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LOEX Option: A Combination of Exchange Option and Lookback Option

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Abstract

In this article, we consider modeling and pricing a combination of two options (Exchange option and Lookback option) that we call the LOEX option. It is a type of exotic option, or clearer, path-dependent option because its price depends on the maximum and minimum prices of the assets to be considered. In reality, we introduce a conditional claim on two assets: the holder can change the first asset with the highest price to the second with the lowest price or cancel the transaction at the strike time. In this paper, after describing and modeling this option, we use numerical methods to estimate the pricing using the MATLAB software and present the results.

Keywords: Exchange option, Lookback option, Black–Scholes Option pricing, Mesh-free method.

1|Introduction

Finance is a huge world. One part of this world is financial instruments, and there are a lot of papers about modeling the pricing of these instruments. In this paper, we consider introducing the" LOEX option." The LOEX option comes with a combination of two options. We designed this security by combining the Exchange option and the Lookback option. In other words, this option comes from applying the Lookback option on the

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Exchange option. The LOEX option is a kind of conditional claim focusing on two assets that consider one with the highest price and the other with the lowest.

The research background in this area is generally divided into two parts because there is no research directly related to this option:

Exchange Option

The Exchange Option is one of the interesting instruments provided by Margrabe (1978)[1]. Several studies have been carried out on the modeling of this security, which can be cited by El Karoui et al. (1995), Antonelli et al. (2010), Cheang and Chiarella (2011), Kim and Koo (2016), and Wang(2016)[2, 3, 4, 5, 6, 7] . El Karoui et al. obtained a close formula for the Exchange Option using the change of numeraire, and Kim and Koo provided a close formula for the state of the two assets being the same drift using a Mellin transform. Antonelli et al. worked on this option considering the other structure of Volatility. Cheang and Chiarella, as well as Wang, modeled the Jump diffusion for this structure.

Lookback option

The first studies in the context of the Lookback option were Goldman et al. (1979) and Conze and Viswanathan (1991) [8, 9]research, which provided the exact solution for the floating and fix strike model. In this regard, Dai et al. (2004) [10] provided a close formula for quanto Lookback options. He et al. (1998) [11] have priced it using numerical methods and Monte Carlo by obtaining a joint density function. Jeon and Yoon (2016) [12] also got a closed formula for put-back options using the Mellin transform.

As we said, in this paper, we are combining the Exchange option and the Lookback option to provide a new option called the LOEX option, and for the first time in this study, modeling and pricing it. In addition to being a risk hedging instrument, this financial derivative can have another function; due to its structure, we can use it as a market stimulus. For example, this option can be used to move the ask queues of an asset.

To achieve the intended purpose, we will introduce the Exchange and Lookback options in the next section and provide the price equation for each. In Section 3, we will use the mathematical concepts of LOEX option modeling, develop a PDE for its price model, and provide a way to solve the PDE. Finally, in Section 4, we will run the model with MATLAB software, compare it with the Exchange option price, and show the results.

2|Preliminary Concepts

In this section, we consider to provide the instruments that made this idea come to our mind. To start, we suppose that (Ω, F, Q) is the probability measure space where Ω is a sample space, F is an event space and Q is a probability function and we work in this space. In the following, we will define the Exchange and Lookback options and show the price equation. To this end, we consider the following two risky assets:

$$\begin{aligned} dS &= rSdt + \sigma_1 S dw_t^1, \\ dS^* &= rS^*dt + \sigma_2 S^* dw_t^2, \end{aligned} \tag{1}$$

where r is the interest rate that is drift of these equations and $\sigma_i, i \in \{1, 2\}$ are the volatility of the assets.

2.1|Exchange Option

An exchange option is a conditional claim that the holder can convert assets to another asset at strike time or NOT. Assume that $C(t, S, S^*)$ is the Exchange option price at time t with the price of assets S and S^* and the payoff function is as follows:

$$C(T, S(T), S^*(T)) = (S(T) - S^*(T))^+. \tag{2}$$

Now, using the Feynman–Kac formula:

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + rS^*\frac{\partial C}{\partial S^*} + \frac{1}{2}\sigma_1^2 S^2 \frac{\partial^2 C}{\partial S^2} + \rho\sigma_1\sigma_2 SS^* \frac{\partial^2 C}{\partial S \partial S^*} + \frac{1}{2}\sigma_2^2 {S^*}^2 \frac{\partial^2 C}{\partial {S^*}^2} - rC = 0 \tag{3}$$

which $\rho dt = dw_t^1 dw_t^2$ and its terminal condition is as follows:

$$C\left(T,S,S^*\right)=\left(S^*-S\right)^+. \tag{4}$$

The closed formula for the Exchange option according to PDE (3) is as follows [5]

$$C(t,S_1,S_2)=S_1\Phi(d)-S_2\Phi(d-\sigma\sqrt{T-t}), \tag{5}$$

where

$$\begin{aligned} d &= \frac{ln\left(\frac{S_1}{S_2}\right)}{\sigma\sqrt{T-t}}+\frac{\sigma\sqrt{T-t}}{2},\\ \sigma &= \sqrt{\sigma_1^2+\sigma_2^2-2\rho\sigma_1\sigma_2},\\ \Phi(x) &= \int_{-\infty}^{x} e^{-0.5x^2}dx. \end{aligned} \tag{6}$$

2.2|Lookback Option

The Lookback option is an exotic option that depends on the asset price path. In general, the Lookback option is divided into four categories. In this section, we summarize everything and present the price equation. Consider the following symbols:

$$\begin{aligned} M(S(t),t) &= \max_{0\le\omega\le t} S(\omega),\\ m(S(t),t) &= \min_{0\le\omega\le t} S(\omega). \end{aligned} \tag{7}$$

Now we have Lookback options and their price equations [13]

Floating strike Lookback call option.

$$\begin{aligned} &payoff \to S(T)-m(S(T),T)\\ &Eq. \to S\Phi\left(\frac{\ln\left(\frac{S}{m(S,t)}\right)+\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)-m(S,t)e^{-r(T-t)}\Phi\left(\frac{\ln\left(\frac{S}{m(S,t)}\right)+\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)\\ &-\frac{S\sigma_1^2}{2r}\Phi\left(\frac{\ln\left(\frac{m(S,t)}{S}\right)-\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)+e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\Phi\left(\frac{\ln\left(\frac{m(S,t)}{S}\right)+\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right). \end{aligned} \tag{8}$$

Floating strike Lookback put option.

$$\begin{aligned} &payoff \to M(S(T),T)-S(T)\\ &Eq. \to -S\Phi\left(-\frac{\ln\left(\frac{S}{M(S,t)}\right)+\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)+M(S,t)e^{-r(T-t)}\Phi\left(-\frac{\ln\left(\frac{S}{M(S,t)}\right)+\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)\\ &+\frac{S\sigma_1^2}{2r}\Phi\left(\frac{\ln\left(\frac{S}{M(S,t)}\right)+\left(r+\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)-e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(\frac{M}{S}\right)^{\frac{2r}{\sigma_1^2}}\Phi\left(\frac{\ln\left(\frac{S}{M(S,t)}\right)-\left(r-\frac{\sigma_1^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right). \end{aligned} \tag{9}$$

Fixed strike Lookback call option.

$$payoff \to MAX(M(S(T),T)-K,0)$$

At the time t, the price would be $e^{-r(T-t)}E[\max(\max(M_0^t(S(t),t),M_t^T(S(T),T))-K,0)]$,

$$\begin{aligned} &Eq. \to \begin{cases} H(t,S;K) & M_0^t(S(t),t)\le K\\ e^{-r(T-t)}(M_0^t(S(t),t)-K)+H(t,S;M_0^t(S(t),t)) & M_0^t(S(t),t)\ge K \end{cases},\\ &H(t,S;N)=S\Phi(d_1)-Ne^{-r(T-t)}\Phi(d_2)+e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(e^{r(T-t)}\Phi(d_1)-\left(\frac{S}{N}\right)^{\frac{-2r}{\sigma_1^2}}\Phi\left(d_1-\frac{2r\sqrt{T-t}}{\sigma}\right)\right). \end{aligned} \tag{10}$$

$$\begin{aligned} d_1 &= \frac{\ln(\frac{S}{N})+(r+\frac{\sigma_1^2}{2})}{\sigma\sqrt{T-t}},\\ d_2 &= \frac{\ln(\frac{S}{N})+(r-\frac{\sigma_1^2}{2})}{\sigma\sqrt{T-t}}. \end{aligned} \tag{11}$$

Fixed strike Lookback put option.

$$payoff \to MAX(K - m(S(T),T),0)$$

At the time t, the price would be $e^{-r(T-t)}E[\max(K - \min(m_0^t(S(t),t), m_t^T(S(T),T)),0)]$,

$$\begin{array}{l} For \quad K \leq m_0^t(S(t),t) \\ Eq. \to Ke^{-r(T-t)}\Phi(-d_2) - S\Phi(-d_1) + e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(-e^{r(T-t)}\Phi(-d_1) - \left(\frac{S}{K}\right)^{\frac{-2r}{\sigma_1^2}}\Phi\left(-d_1 + \frac{2r\sqrt{T-t}}{\sigma}\right)\right), \\ For \quad K \geq m_0^t(S(t),t) \\ Eq. \to e^{-r(T-t)}(K + m_0^t(S(t),t)(\Phi(d_2)-1)) - S\Phi(-d_1), \\ \qquad + e^{-r(T-t)}\frac{S\sigma_1^2}{2r}\left(e^{r(T-t)}\Phi(-d_1) + \left(\frac{S}{m_0^t(S(t),t)}\right)^{\frac{-2r}{\sigma_1^2}}\Phi\left(-d_1 + \frac{2r\sqrt{T-t}}{\sigma_1}\right)\right). \end{array} \tag{12}$$

d_1 and d_2 are the same as (11) formula.

3|LOEX Option

In this section, we model and price the LOEX option using the concepts described in the previous section. As stated, this option is made by implementing the Lookback option on the Exchange option. The holder can replace an asset with the highest price from the start of the contract to maturity time with the lowest price of another asset in that period. Since the price of this option depends on the asset price path, this option is an exotic option.

To begin, we consider formula (1) and the following symbols:

$$\begin{array}{l} M(S(t),t) = \max\limits_{0\leq\omega\leq t} S(\omega), \\ m(S^*(t),t) = \min\limits_{0\leq\omega\leq t} S^*(\omega). \end{array} \tag{13}$$

Now assume $G(t,S,S^*)$ is the price of a LOEX option at time t and the price of assets S and S^*, the payoff corresponding to this option is as follow:

$$G(T,S(T),S^*(T)) = (M(S(T),T) - m(S^*(T),T))^+. \tag{14}$$

Also, the price of this option at t, due to the Q martingale measure is as follow:

$$\begin{array}{l} G(t,S,S^*) = e^{-r(T-t)}E^Q[G(T,S(T),S^*(T))|M(S(t),t),m(S^*(t),t)] = \\ e^{-r(T-t)}E[(M(S(T),T) - m(S(T),T))^+|M(S(t),t),m(S^*(t),t)]. \end{array} \tag{15}$$

The LOEX option price differential with the Ito lemma will follow the following SDE:

$$\begin{array}{l} dG = \left(G_t + \alpha_1 SG_S + \alpha_2 S^* G_{S^*} + \sigma_1^2 S^2 G_{SS} + \sigma_2^2 {S^*}^2 G_{S^*S^*} + \rho\sigma_1\sigma_2 SS^* G_{SS^*}\right)dt \\ \quad + (\sigma_1 SG_S + \sigma_2 S^* G_{S^*})\,dW. \end{array} \tag{16}$$

Now we use the Feynman–Kac formula:

$$\frac{\partial G}{\partial t} + rS\frac{\partial G}{\partial S} + rS^*\frac{\partial G}{\partial S^*} + \frac{1}{2}\sigma_1^2 S^2\frac{\partial^2 G}{\partial S^2} + \rho\sigma_1\sigma_2 SS^*\frac{\partial^2 G}{\partial S\partial S^*} + \frac{1}{2}\sigma_2^2 {S^*}^2\frac{\partial^2 G}{\partial {S^*}^2} - rG = 0, \tag{17}$$

that its terminal condition is as follows:

$$G(T,S(T),S^*(T)) = (M(S(T),T) - m(S^*(T),T))^+. \tag{18}$$

PDE solution

Sometimes, an issue is a problem that we cannot access the close form of problems, so the numerical methods help us change this circumstance. There are a lot of numerical methods that are used for these problems [14] [15]. In this solution section, we consider solving PDE (17) by numerical methods. The solution method used for the PDE (17) is the mesh-free numerical approximation method [16, 17]. Therefore, by property of exchange and lookback options, we consider boundary and initial conditions as follows:

$$\begin{aligned}
&G(0,S,S^*) = (S-S^*)^+,\\
&G(\tau,0,S^*) = 0,\\
&G(\tau,S,0) = S,\\
&\lim_{S,S^*\to\infty} G(\tau,S,S^*) = \lim_{S,S^*\to\infty} \tfrac{S}{\psi^{S^*}},
\end{aligned} \tag{19}$$

where ψ is a large number.

For applying the meshfree numerical approximation method, suppose the change of assets S and $S*$ is in ithe interval $[0, S_{max}]$ and $[0, S^*_{max}]$ respectively, and also discrete the first interval in M points and the second N points so we have the following radial basis function:

$$G(t,S,S^*) = \sum_{i=1}^{MN} \lambda_i(t)\varphi(S,S^*), \tag{20}$$

where

$$\varphi(x,y) = \sqrt{x^2+y^2+\varepsilon^2}, \tag{21}$$

and λ is a function that depends on time.

We rewrite the PDE's (17) derivatives as follow:

$$\begin{aligned}
&G_{S^{(*)}}(t,S,S^*) = \sum_{i=1}^{MN} \lambda_i(t)\varphi_{S^{(*)}}(S,S^*) =: \varphi_{S^{(*)}}\lambda,\\
&G_{S^{(*)}S^{(*)}}(t,S,S^*) = \sum_{i=1}^{MN} \lambda_i(t)\varphi_{S^{(*)}S^{(*)}}(S,S^*) =: \varphi_{S^{(*)}S^{(*)}}\lambda,\\
&G_t(t,S,S^*) = \sum_{i=1}^{MN} \dot{\lambda}_i(t)\varphi(S,S^*) =: \varphi\dot{\lambda},
\end{aligned} \tag{22}$$

where $\varphi, \varphi_{S^{(*)}}, \varphi_{S^{(*)}S^{(*)}}$supposed that are matrix as follow:

$$\begin{aligned}
&\varphi_{i,j} = \varphi(S_i, S^*_j),\\
&\varphi_{S^{(*)}}{}_{i,j} = \varphi_{S^{(*)}}(S_i, S^*_j),\\
&\varphi_{S^{(*)}S^{(*)}}{}_{i,j} = \varphi_{S^{(*)}S^{(*)}}(S_i, S^*_j).
\end{aligned} \tag{23}$$

Then, we could have the PDE (17) as below:

$$\varphi\dot{\lambda} + L\lambda = 0, \tag{24}$$

where

$$L = rS\varphi_S + rS^*\varphi_{S^*} + \frac{1}{2}\sigma_1^2 S^2\varphi_{SS} + \rho\sigma_1\sigma_2 SS^*\varphi_{SS^*} + \frac{1}{2}\sigma_2^2 {S^*}^2\varphi_{S^*S^*} - r\varphi. \tag{25}$$

Using the concept of derivative, suppose the interval of time is $[0,T]$ whereas mentioned, T is strike time and consider K as the number of equal discrete points of this interval with dt difference so

$$\dot{\lambda} = \frac{\lambda(t_{k+1}) - \lambda(t_k)}{dt} =: \frac{\lambda^{k+1} - \lambda^k}{dt} \tag{26}$$

and using the numerical θ-method, we can rewrite the ODE (24) as follows:

$$A\lambda^{k+1} = B\lambda^k, \tag{27}$$

where

$$\begin{aligned}
&A = \varphi + \theta dtL,\\
&B = \varphi - (1-\theta)dtL,\\
&1 \le k \le K-1.
\end{aligned} \tag{28}$$

Algorithm

1. Using initial condition, obtain $\lambda^k, k = 1$;
2. Compute $A\lambda^{k+1} = B\lambda^k$;
3. $G = \varphi\lambda^k$;
4. Apply boundary condition on G;
5. $\lambda^{k+1} = \varphi^{-1}G, k = k + 1$, and if $k < K$ go step 2.

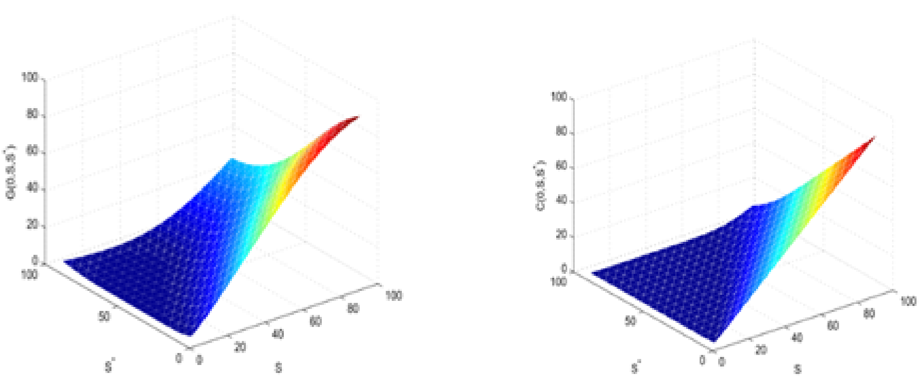
4|Numerical Result

In this section, we use the solving method in section (3.1) and the Kim and Koo (2016) data to implement the LOEX option that σ_1, $\sigma_2 = 0.125$, $\rho = -0.35$, $r = 0.05$ and finally, we compare our model with their price model.

TABLE 1. Variables

Variable	Value
S_{max}, S^*_{max}	100
T	1
M, N	20
ψ	1.5
dt	0.01

Using the MATLAB software, the LOEX option price curve will be as figure (1.A), and with the same data and the Kim and Koo (2016) price formula, we get the Exchange option price curve as figure (1. B).



(A) LOEX option price curve

(B) Exchange option price curve

FIGURE 1. Option price curves

In Figure (2), we consider presenting the Discrepancy between LOEX option and the Exchange option as follows:

In the figure (2), there is a huge difference between the LOEX option and the Exchange option. The price changes of the LOEX option are more than the price changes of the Exchange option. According to this option's structure, this difference's existence was not unpredictable. In the following, we can observe the Discrepancy of sensitivity of two options according to asset price differential in figure (3).

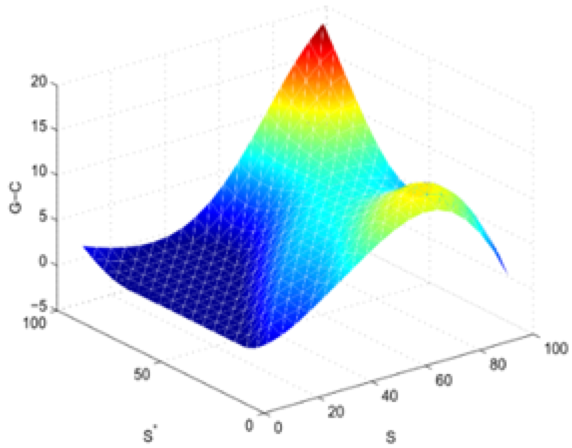
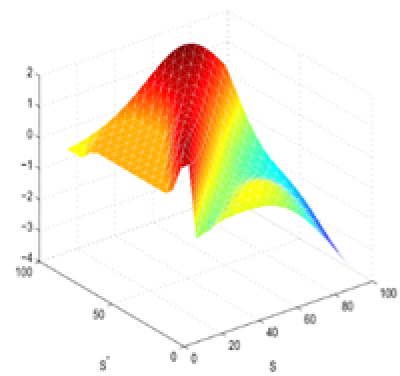
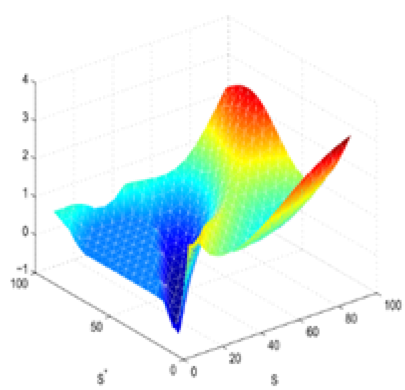


FIGURE 2. Difference between options



(A) $\frac{\partial G}{\partial S} - \frac{\partial C}{\partial S}$



(B) $\frac{\partial G}{\partial S^*} - \frac{\partial C}{\partial S^*}$

FIGURE 3. Discrepancy of sensitivity of two options

5|Conclusion

In this article, we have presented a new option called the LOEX option. In fact, in this study, we introduced the new option by combining the Exchange and Lookback options. In this way, we have modeled this option, solved it by numerical methods, and provided it with the MATLAB software. Finally, after obtaining the price of the Exchange option, we have compared it with the LOEX option curve. The price difference figure shows a significant difference between the two assets. Due to the LOEX option's dependency on the path, this option is much more expensive than the Exchange option, and this difference increases with the increase in the price of the two assets.

Furthermore, we can develop the model and make it more realistic for the market. For example, we can add components like credit risk and improve the performance of the price model.

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Author Contribution

R. Ghasem Pour: methodology, software, and editing. S.P. Azizi: conceptualization. S.A.Waloo: writing and editing. All authors have read and agreed to the published version of the manuscript.

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Conflicts of Interest

The authors declare that there is no conflict of interest concerning the reported research findings. Funders played no role in the study's design, in the collection, analysis, or interpretation of the data, in the writing of the manuscript, or in the decision to publish the results.

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