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Tripolar Fuzzy Ideals In CV Algebra

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Abstract

In this work the concept of CV-algebra is introduced and some of its properties like, self-distributiveness, essence, ideals, fuzzy ideals and tripolar fuzzy ideals are investigated. Homomorphism between CV-algebras is defined and it is established that images and pre-images of essences are also essence when the homomorphism is onto.

Keywords: Self-distributive, Essence, Sub algebra, Homomorphism, Fuzzy ideal, Tripolar set.

1|Introduction

It was Zadeh who introduced the notion of fuzzy set in 1965, as a generalization of the concept of characteristic function of a set. Fuzzy set is identified as a better tool for the scientific study of uncertainty, and came as a boost to the researchers working in the field of uncertainty. Many extensions and generalizations of fuzzy set was conceived by a number of researchers and a large number of real-life applications were developed in a variety of areas like, logic, finite state machines, automata theory, artificial intelligence, computer science, control engineering and so on. In addition to this, parallel analysis of the classical results of many branches of Mathematics was undertaken in the fuzzy settings. One such abstract area in the branch of fuzzy algebra. It was initiated by Rosenfeld, who coined the idea of fuzzy subgroup of a group in 1971 and studied some basic properties of this structure. Iseki and Tanaka have introduced the theory of BCK-algebras. Iseki has introduced BCI-algebras. Hu and Li introduced BCH-algebras. The class of BCI-algebras is a proper subclass of the class

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of BCH-algebras.
Zhang introduced the concept of bipolar fuzzy set as an extension of fuzzy set whose membership degree range is [-1,1]. In a bipolar valued fuzzy set the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degrees on (0,1] indicate that elements somewhat satisfy the property, and the membership degrees on [-1,0) indicate that elements somewhat satisfy the implicit counter property. Kim and Kim introduced the notion of a BE-algebra. Ahn and So introduced the notion of ideals in BE-algebras. Kim has studied the notion of essence in BE algebra. The monograph by Chinnadurai gives a detailed discussion on fuzzy ideals in algebraic structures. Senapti et al. introduced and investigated some of their properties of fuzzy dot subalgebras, fuzzy normal dot subalgebras and fuzzy dot ideals of B-algebras. The notion of cubic intuitionistic q-ideals in BCI-algebras is introduced by Senapti et al. Murali Krishna Rao introduced tripolar fuzzy interior ideals of Γ- semigroup. Jana et al initiated the concept of bipolar fuzzy soft subalgebras and ideals of BCK/BCI-algebras baesd on bipolar fuzzy points. Mostafa1 and Ghanem introduced tripolar fuzzy sub implicative ideals of KU-Algebras. In this research work, we introduce the concept of CV-algebra and tripolar fuzzy ideals in CV-algebra.

2|CV-algebra

Definition 2.1. A non-empty set U with a binary operation $\star$ satisfying the conditions

(i) $u \star 0 = 0 = 0 \star u$

(ii) $u \star u = u$

(iii) $u \star (v \star w) = w \star (u \star v)$, for all $u, v, w \in U$ is called CV-algebra.

Example 2.1. The set $U = \{0, 1, 2, 3\}$ with the binary operation $\star$, with the following Cayley table becomes CV-algebra.

$\star$	0	1	2	3
0	0	0	0	0
1	0	1	2	1
2	0	1	2	3
3	0	1	1	3

Definition 2.2. A CV-algebra $(U, \star, 0)$ is said to be self-distributive if
$u \star (v \star w) = (u \star v) \star (u \star w)$ for all $u, v, w \in U$.

Example 2.2. The set $U = \{0, 1, 2, 3\}$ with the binary operation $\star$, with the following Cayley tables become self-distributive CV-algebra.

$\star$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	1	2	3
3	0	2	2	3

and

$\star$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	1	2	3
3	0	1	2	3

Definition 2.3. If a non-empty subset E of the CV-algebra U satisfies the condition $U \star E = E$, then E is called an essence of U.

Example 2.3. In Example 2.2. $\{0\}$ and U itself are essences of U. $E_1 = \{0, 1\}, E_2 = \{0, 1, 2\}, E_3 = \{0, 1, 3\}$ are also an essences of U but $E_4 = \{0, 2\}, E_5 = \{0, 3\}$ are not an essences of U.

Lemma 2.1. If $(U, \star, 0)$ be CV-algebra, then $u \star (v \star u) = v \star u$.
Proof: $u \star (v \star u) = v \star (u \star u) = v \star u$, by (iii) and (ii) of Definition 2.1.

Lemma 2.2. If $(U, \star, 0)$ be CV-algebra, then

(i) $u \star (0 \star u) = 0$ and (ii) $u \star (w \star u) = w \star u$.
Proof:

(i) $u \star (0 \star u) = u \star 0 = 0$, by (i) and (ii) of Definition 2.1.

(ii) $u \star (w \star u) = w \star (u \star u) = w \star u$, by (iii) and (ii) of Definition 2.1.

Lemma 2.2. Every essence contains the element 0.
Proof: Let E be an essence of U. Then $\emptyset \neq E = U \star E$, and so there exists $a \in E$ such that and thus $0 = U \star 0 = U \star E = E$. Hence every essence contains the element 0.

Theorem 2.1. Every essence is a sub algebra of U.
Proof: Let E be an essence of U and let $u, v \in E$. Then $u \star v \in E \star E \subseteq U \star E = E$. And so E is a sub algebra of U.
But the converse is not true. In Example 2.1. the set $\{0, 2\}$ is a sub algebra but not an essence of U.

Theorem 2.2. Let E and F be essences of U. Then $E \cap F$ and $E \cup F$ are an essences of U.
Proof: Let $R = E \cap F$. Then $0 \star R \subseteq U \star R = U \star (E \cap F) = (U \star E) \cap (U \star F) = E \cap F$.
So $U \star R = R$ is an essence of U.
Now let $S = E \cup F$. Then $0 \star S \subseteq U \star S = U \star (E \cup F) = (U \star E) \cup (U \star F) = E \cup F$.
So $U \star S = S$ is an essence of U.

Lemma 2.3. Let E be an essence of CV-algebra of U. If $0 \in F \subseteq U$, then $F \star E = E$.
Proof: Let E be an essence of CV-algebra of U. Then $0 \star E \subseteq F \star E \subseteq U \star E = E$. Therefore $F \star E = E$.

Lemma 2.4. Let U be a CV-algebra. If $0 \in E \subseteq U$, then F is contained in $E \star F$ for every sub set F of U.
Proof: Let $f \in F$. Then $f = 0 \star f \in E \star F$ and so F is contained in $E \star F$.

Theorem 2.3. Let U be a CV-algebra. If E is an essence of U and F is a sub algebra of U and $E \subseteq F$, then $E \cup F$ and $E \cap F$ are the essences of F.
Proof: $F \star (E \cap F) = (F \star E) \cap (F \star F) \subseteq (U \star E) \cap F \subseteq E \cap F$.
Obviously, $E \cap F \subseteq F \star (E \cap F)$. Therefore $F \star (E \cap F) = E \cap F$.
Hence $E \cap F$ is an essence of F.
Similarly $E \cup F$ is also an essence of F if $E \subseteq F$.

Definition 2.4. Let U and V be CV-algebras. A mapping $\varphi : U \longrightarrow V$ is called a homomorphism if $\varphi(u \star v) = \varphi(u) \star \varphi(v)$ for all $u, v \in U$. Note that $\varphi(0) = 0$.

Lemma 2.5. For any sub sets E, F and G of CV-algebra U, we have

(1) $E \subseteq F \Rightarrow E \star G \subseteq F \star G, G \star E \subseteq G \star F$,

(2) $(E \cap F) \star G \subseteq (E \star G) \cap (F \star G)$,

(3) $G \star (E \cap F) \subseteq (G \star E) \cap (G \star F)$,

(4) $(E \cup F) \star G = (E \star G) \cup (F \star G)$,

(5) $G \star (E \cup F) = (G \star E) \cup (G \star F)$.
Proof:

(1) Let $u \in E \star G$. Then $u = e \star g$ for some $e \in E$ and $g \in G$. Since $E \subseteq F$, it follows that $u = e \star g$ for some $e \in F$ and $ginG$. So that $u \in F \star G$. Which implies that $E \star G \subseteq F \star G$. Similarly we can obtain $G \star E \subseteq G \star F$.

(2) Since $E \cap F \subseteq E$ and $E \cap F \subseteq F$. From $u \in E \star G$, we get $(E \cap F) \star G \subseteq E \star G$ and $(E \cap F) \star G \subseteq F \star G$, and so $(E \cap F) \star G \subseteq (E \star G) \cap (F \star G)$.

(3) From (1)and (2), we can write $G \star (E \cap F) \subseteq (G \star E) \cap (G \star F)$.

(4) Since $E \subseteq E \cup F$ and $F \subseteq E \cup F$. We get $E \star G \subseteq (E \cup F) \star G$ and $F \star G \subseteq (E \cup F) \star G$, and so $(E \star G) \cup (F \star G) \subseteq (E \cup F) \star G$. If $u \in (E \cup F) \star G$, then $u = v \star g$ for some $v \in E \cup F$ and $g \in G$. It follows that $u = v \star g$ for some $v \in E$ and $g \in G$ or $u = v \star g$ for some $v \in F$ and $g \in G$. So that $u = v \star g \in E \star G$ or $u = v \star g \in F \star G$. Hence $u \in (E \star G) \cup (F \star G)$, which shows that $(E \cup F) \star G \subseteq (E \star G) \cup (F \star G)$. Thus $(E \cup F) \star G = (E \star G) \cup (F \star G)$.

(5) From (1)and (4), we can write $G \star (E \cup F) = (G \star E) \cup (G \star F)$.

Lemma 2.6. For any subsets E, F of CV-algebra U, we have the following results

(1) $E^{c^c} = E$,

(2) $(E \cup F)^c = E^c \cap F^c$ and $(E \cap F)^c = E^c \cup F^c$,

(3) $E \cup F = F \cup E$ and $E \cap F = F \cap E$,

(4) $E \cup (F \cup G) = (E \cup F) \cup G$ and $E \cap (F \cap G) = (E \cap F) \cap G$,

(5) $E \cup \emptyset = E$ and $E \cap U = E$,

(6) $E \cup E^c = U$ and $E \cap E^c = \emptyset$.

Proof:

(1) Let $u \in E \Rightarrow u \notin E^c \Rightarrow u \in E^{c^c} \Rightarrow E \subseteq E^{c^c}$. Now let $u \in E^{c^c} \Rightarrow u \notin E^c \Rightarrow u \in E \Rightarrow E^{c^c} \subseteq E$. Hence we get $E = E^{c^c}$.

(2) Let $u \in (E \cup F)^c \Leftrightarrow u \notin E \cup F \Leftrightarrow u \notin E$ and $u \notin F \Leftrightarrow u \in E^c$ and $u \in F^c \Leftrightarrow u \in E^c \cap F^c$. Then $(E \cup F)^c = E^c \cap F^c$. And let $u \in (E \cap F)^c \Leftrightarrow u \notin E \cap F \Leftrightarrow u \notin E$ or $u \notin F \Leftrightarrow u \in E^c$ or $u \in F^c \Leftrightarrow u \in E^c \cup F^c$. Then $(E \cap F)^c = E^c \cup F^c$.

(3) $E \cup F = \{u : u \in E \text{ or } u \in F\} = \{u : u \in F \text{ or } u \in E\} = F \cup E$ and $E \cap F = \{u : u \in E$ and $u \in F\} = \{u : u \in F \text{ and } u \in E\} = F \cap E$.

(4) Let $u \in E \cup (F \cup G) \Leftrightarrow u \in E$ or $u \in F \cup G \Leftrightarrow u \in E$ or $u \in F$ or $u \in G \Leftrightarrow u \in (E$ or $F)$ or $u \in G \Leftrightarrow u \in (E \cup F) \cup G$. Then $E \cup (F \cup G) = (E \cup F) \cup G$. And also let $u \in E \cap (F \cap G) \Leftrightarrow u \in E$ and $u \in F \cap G \Leftrightarrow u \in E$ and $u \in F$ and $u \in G \Leftrightarrow u \in (E$ and $F)$ and $u \in G \Leftrightarrow u \in (E \cap F) \cap G$. Then $E \cap (F \cup G) = (E \cap F) \cap G$.

(5) Let $u \in E \cup \emptyset \Leftrightarrow u \in E$ or $u \in \emptyset \Leftrightarrow u \in E$. Then $E \cup \emptyset = E$. And also let $u \in E \cap U \Leftrightarrow u \in E$ and $u \in U \Leftrightarrow u \in E$. Then $E \cap U = E$.

(6) Let $u \in E \cup E^c \Leftrightarrow u \in E$ or $u \in E^c \Leftrightarrow u \in U$. Then $E \cup E^c = U$. And also let $u \in E \cap E^c \Leftrightarrow u \in E$ and $u \in E^c \Leftrightarrow u \in \emptyset$. Then $E \cap E^c = \emptyset$.

Theorem 2.4. Let $\varphi : U \longrightarrow V$ be a homomorphism of CV-algebra.

(1) If φ is onto and E is an essence of U, then $\varphi(E)$ is an essence of V,

(2) If F is an essence of V, then $\varphi^{-1}(F)$ is an essence of U.

Proof:

(1) Suppose that φ is onto and E is an essence of U.
Let $f \in \varphi(E)$ and $v \in V$. Then $f = \varphi(e)$ and $v = \varphi(u)$ for some $e \in E$ and $u \in U$.
Thus $v \star f = \varphi(u) \star \varphi(e) = \varphi(u \star e) \in \varphi(U \star E) = \varphi(E)$ and so $V \star \varphi(E) \subseteq \varphi(E)$. Obviously $\varphi(E) \subseteq V \star \varphi(E)$.
Therefore $V \star \varphi(E) = \varphi(E)$.
Hence $\varphi(E)$ is an essence of V.

(2) Let $e \in \varphi^{-1}(F)$ and $u \in U$. Then $\varphi(e) \in F$ and $\varphi(u) \in V$. $\varphi(u \star e) = \varphi(u) \star \varphi(e) \in V \star F = F$. We have $u \star e \in \varphi^{-1}(F)$. That is, $U \star \varphi^{-1}(F) \subseteq \varphi^{-1}(F)$. Obviously $\varphi^{-1}(F) \subseteq U \star \varphi^{-1}(F)$.
Therefore $U \star \varphi^{-1}(F) = \varphi^{-1}(F)$.
Hence $\varphi^{-1}(F)$ is an essence of U.

Theorem 2.5. Let E be a CV-algebra, then $E \cup E^c$ is also a CV-algebra.

Proof: Given E is a CV-algebra, then there exists $0 \in E$. But $0 \notin E^c$. Definitely $0 \in E \cup E^c$. Hence $E \cup E^c$ is also a CV-algebra.

Lemma 2.6. Let E, F be essences of CV-algebra of U. Then $U \star E = U \star F$ iff $E = F$.
Proof: Since by Definition 2.5. if $U \star E = U \star F$, then $E = F$.
If we assume $E = F$, then $E \subseteq F$ and $F \subseteq E \Rightarrow U \star E \subseteq U \star F$ and $U \star F \subseteq U \star E \Rightarrow U \star E = U \star F$.

3|On ideals in CV-algebra

Definition 3.1. A non-empty subset $\mathcal{I}$ of U is called an ideal of U if

(i) If $u \in U$ and $l \in \mathcal{I}$, then $u \star l \in \mathcal{I}$, i.e., $U \star \mathcal{I} \subseteq \mathcal{I}$,

(ii) If $u \in U$ and $l, m \in \mathcal{I}$, then $(l \star (m \star u)) \star u \in \mathcal{I}$.

Example 3.1. In Example 2.2. $\{0\}$ and U are the only ideals, since
$(1 \star (1 \star 2)) \star 2 = 2 \neq \{0, 1\}$ and $(2 \star (2 \star 3)) \star 3 = 3 \neq \{0, 1, 2\}$.

Lemma 3.1. Every ideal of U is an essence of U.
Proof: In Example 2.2. $\{0\}$ and U are the only ideals of U that are also an essence of U. But every essence of U need not be an ideal of U.

Theorem 3.1. Every ideal of U contains 0.
Proof: Let $\mathcal{I} \neq \emptyset$ be an ideal of U. There exists $u \in \mathcal{I}$. Then
$0 = 0 \star u \in U \star \mathcal{I} \subseteq \mathcal{I}$.
Hence $0 \in \mathcal{I}$.

Theorem 3.2. If $\mathcal{I}$ is an ideal of U, then $(l \star u) \star u \in \mathcal{I}$ for all $l \in \mathcal{I}$ and $u \in U$.
Proof: Let $m = u$ in Definition 3.1(ii). Thus, $(l \star (u \star u)) \star u = (l \star u) \star u \in \mathcal{I}$.

Lemma 3.2. Let $\mathcal{I}$ be an ideal of U. If $l \in \mathcal{I}$ and $l \geq u$, then $u \in \mathcal{I}$.
Proof: Let $l \in \mathcal{I}, u \in U$ with $l \geq u$. Then $l \star u = u$. Hence $u = u \star u = (l \star u) \star u \in \mathcal{I}$. Therefore $u \in \mathcal{I}$.

Definition 3.2. Let $\mathcal{I}$ be an ideal of U. Define $\mathcal{I}_t$ by
$\mathcal{I}_t = \{u \in U | t \star u \in \mathcal{I}\}$ for any $t \in U$.

Lemma 3.3. Let U be a self-distributive CV-algebra and $\mathcal{I}$ an ideal of U. Then $\mathcal{I}_t$ is a sub algebra of the CV-algebra U.
Proof: Let $l, m \in \mathcal{I}_t$. Then $t \star l \in \mathcal{I}$ and $t \star m \in \mathcal{I}$. By self-distributive property
$t \star (l \star m) = (t \star l) \star (t \star m) \subseteq \mathcal{I} \star \mathcal{I} \subseteq U \star \mathcal{I} \subseteq \mathcal{I}$.
Then $l \star m \in \mathcal{I}_t$.
Therefore $\mathcal{I}_t$ is a sub algebra of the CV-algebra U.

Theorem 3.3. Let U be a self-distributive CV-algebra and $\mathcal{I}$ is an ideal of U. Then $\mathcal{I}_t$ is an ideal of the CV-algebra U.
Proof: Let $u \in U$ and $l \in \mathcal{I}_t$. We have $t \star l \in \mathcal{I}$, and so $t \star (u \star l) = (t \star u) \star (t \star l) \in U \star \mathcal{I} \subseteq \mathcal{I}$.
Then $u \star l \in \mathcal{I}_t$. $\dashrightarrow$ (1)
Also let $l, m \in \mathcal{I}_t$ and $u \in U$. We obtain $t \star l \in \mathcal{I}$ and $t \star m \in \mathcal{I}$.
Then $t \star ((l \star (m \star u)) \star u) = (t \star (l \star (m \star u))) \star (t \star u) = ((t \star l) \star (t \star (m \star u))) \star (t \star u) = ((t \star l) \star ((t \star m) \star (t \star u))) \star t \star u) \in \mathcal{I}$.
Then $(l \star (m \star u)) \star u \in \mathcal{I}_t$. $\dashrightarrow$ (2)
From (1) and (2) $\mathcal{I}_t$ is an ideal of the CV- algebra U.

4|Fuzzy ideals in CV-algebra

Definition 4.1. A fuzzy set μ in U is called a fuzzy ideal of U if it satisfies

(i) $\mu(u \star v) \geq \mu(u)$, $\forall u, v \in U$,

(ii) $\mu((u \star (v \star w)) \star w) \geq min\{\mu(u), \mu(v)\}, \forall u, v, w \in U$.

Example 4.1. Define a fuzzy set $\mu : U \longrightarrow [0,1]$ by $\mu(0) = 0.7, \mu(1) = 0.5, \mu(2) = 0.5, \mu(3) = 0.5$ in Example 2.2. Then we have a fuzzy ideal of the CV-algebra.

Lemma 4.1. Every fuzzy ideal μ of U satisfies the inequality: $\mu(0) \geq \mu(u), \forall u \in U$.
Proof: By the definition of CV-algebra and fuzzy ideal in U,

$$\mu(0) = \mu(u \star 0) \geq \mu(u).$$

Lemma 4.2. If μ is a fuzzy ideal of U, then $\mu((u \star 0) \star v) \geq \mu(u), \forall u, v \in U$.
Proof: Since by the definition of fuzzy ideals in U and by Lemma 4.3.
$\mu((u \star 0) \star v) = \mu((u \star (0 \star v)) \star v)$
$\geq min\{\mu(u), \mu(0)\}$
$= \mu(u)$.

For every $l, m \in U$, let μ_l^m be a fuzzy set in U defined by

$$\mu_l^m(u) = \begin{cases} \alpha, & \text{if } l \star (m \star u) = 0 \\ \beta, & otherwise \end{cases}$$

for all $u \in U$ and $\alpha, \beta \in [0,1]$ with $\alpha > \beta$.

Example 4.2. Let $U = \{0, l, m, n\}$ be the CV-algebra with binary operation as defined by the cayley table,

$\star$	0	l	m	n
0	0	0	0	0
l	0	l	m	l
m	0	l	m	n
n	0	l	l	n

Then μ_0^l is not a fuzzy ideal of U.
Proof: $\mu_0^l((l \star (l \star m)) \star m) = \mu_0^l((l \star m) \star m)$
$=\mu_0^l(m \star m)$
$=\mu_0^l(m)$
$=\beta$
$< \alpha$
$=\mu_0^l(l)$
$< min\{\mu_0^l(l), \mu_0^l(l)\}$.
Therefore μ_0^l is not a fuzzy ideal of U.

Theorem 4.1. If U is self-distributive, then the fuzzy set μ_l^m in U is a fuzzy ideal of U for all $l, m \in U$.
Proof: Let $l, m \in U$. For any $u, v \in U$, if $(l \star (m \star v)) \neq 0$, then $\mu_l^m(u) = \beta \leq \mu_l^m(u \star v)$.
Assume that $(l \star (m \star u)) = 0$. Then,
$l \star (m \star (u \star v)) = l \star ((m \star u) \star (m \star v))$
$= (l \star (m \star u)) \star (l \star (m \star v))$
$= 0 \star (l \star (m \star v))$
$= 0$,
and so, $\mu_l^m(u \star v) = \alpha = \mu_l^m(u)$.
Hence $\mu_l^m(u \star v) \geq \mu_l^m(u)$ for all $u, v \in U$.

Now for every $u, v, w \in U$, if $l \star (m \star u) \neq 0$ or $l \star (m \star v) \neq 0$, then $\mu_l^m(u) = \beta$ or $\mu_l^m(v) = \beta$.
Thus $\mu_l^m((u \star (v \star w)) \star w) \geq \beta = min\{\mu_l^m(u), \mu_l^m(v)\}$.
Suppose that $(l \star (m \star u)) = 0$ and $(l \star (m \star v)) = 0$, then
$l \star (m \star (u \star (v \star w)) \star w)) = l \star (m \star (u \star (v \star w)) \star (m \star w)))$
$= l \star ((m \star u) \star (m \star (v \star w)) \star (m \star w))$
$= (l \star (m \star u)) \star (l \star (m \star (v \star w))) \star (l \star (m \star w))$
$= 0 \star ((l \star (m \star v) \star (l \star (m \star w)) \star (l \star (m \star w))$
$= 0 \star 0 \star (l \star (m \star w)) \star (l \star (m \star w))$
$= 0.$
Which implies that, $\mu_l^m((u \star (v \star w)) \star w) = \alpha = min\{\mu_l^m(u), \mu_l^m(v)\}$.
Therefore $\mu_l^m((u \star (v \star w)) \star w) \geq min\{\mu_l^m(u), \mu_l^m(v)\}$.
Hence μ_l^m is a fuzzy ideal of U, $\forall\ l, m \in U$.

Theorem 4.2. The intersection of any set of fuzzy ideals of CV-algebra U is also a fuzzy ideal.
Proof: Let μ_i be a family of fuzzy ideals of CV-algebra U, then for any $u, v, w \in U$,

$(i)(\cap\mu_i)(u \star v) = inf(\mu_i(u \star v))$
$\geq inf(\mu_i(u))$
$= (\cap\mu_i)(u).$

$(ii)(\cap\mu_i)((u \star (v \star w)) \star w) = inf\ \mu_i(((u \star (v \star w)) \star w)$
$\geq inf(min\{\mu_i(u), \mu_i(v)\})$
$= min\{inf(\mu_i(u), inf(\mu_i(v))\}$
$= min\{(\cap\mu_i(u)), (\cap\mu_i(v))\}.$
Hence the intersection of any set of fuzzy ideals of CV-algebra U is also a fuzzy ideal.

5|Tripolar fuzzy ideals in CV-algebra

Definition 5.1. A tripolar fuzzy set $A = \{(u, \mu^+(u), \mu^-(u), \lambda^+(u) | u \in U\}$ in U is called a tripolar fuzzy ideal of U if it satisfies

(i) $\mu^+(u \star v) \geq \mu^+(u)$, $\mu^-(u \star v) \leq \mu^-(u)$ and $\lambda^+(u \star v) \leq \lambda^+(u)\ \forall u, v \in U$,

(ii) $\mu^+((u \star (v \star w)) \star w) \geq min\{\mu^+(u), \mu^+(v)\}$, $\mu^-((u \star (v \star w)) \star w) \leq max\{\mu^-(u), \mu^-(v)\}$ and $\lambda^+((u \star (v \star w)) \star w) \leq max\{\lambda^+(u), \lambda^+(v)\}$, $\forall u, v, w \in U$.

Example 5.1. The set $U = \{0, 1, 2, 3\}$ with the binary operation $\star$, with the following Cayley table becomes tripolar fuzzy ideal of CV-algebra.

$\star$	0	1	2	3
μ^+	0.8	0.5	0.5	0.5
μ^-	0.3	0.5	0.6	0.8
λ^+	0.2	0.4	0.6	0.7

Lemma 5.1. Every tripolar fuzzy ideal A of U satisfies the inequality: $\mu^+(0) \geq \mu^+(u)$, $\mu^-(0) \leq \mu^-(u)$, $\lambda^+(0) \leq \lambda^+(u)\ \forall u \in U$.
Proof: By the definition of CV-algebra and tripolar fuzzy ideal in U,

$$\mu^+(0) = \mu^+(u \star 0) \geq \mu^+(u),\ \mu^-(0) = \mu^-(u \star 0) \leq \mu^-(u),\ \lambda^+(0) = \lambda^+(u \star 0) \leq \lambda^+(u).$$

Hence A satisfies $\mu^+(0) \geq \mu^+(u)$, $\mu^-(0) \leq \mu^-(u)$, $\lambda^+(0) \leq \lambda^+(u)\ \forall u \in U$.

Lemma 5.2. If A is a tripolar fuzzy ideal of U, then $\mu^+((u \star 0) \star v) \geq \mu^+(u)$, $\mu^-((u \star 0) \star v) \leq \mu^-(u)$, $\lambda^+((u \star 0) \star v) \leq \lambda^+(u)$, $\forall u, v \in U$.
Proof: Since by the definition of CV-algebra, tripolar fuzzy ideals in U and by Lemma 5.3.

$\mu^+((u \star 0) \star v) = \mu^+((u \star (0 \star v)) \star v)$
$\geq min\{\mu^+(u), \mu^+(0)\}$
$= \mu^+(u).$
$\mu^-((u \star 0) \star v) = \mu^-((u \star (0 \star v)) \star v)$
$\leq max\{\mu^-(u), \mu^-(0)\}$
$= \mu^-(u).$
$\lambda^+((u \star 0) \star v) = \lambda^+((u \star (0 \star v)) \star v)$
$\leq max\{\lambda^+(u), \lambda^+(0)\}$
$= \lambda^+(u).$

Theorem 5.1. The intersection of any set of tripolar fuzzy ideals of CV-algebra U is also a tripolar fuzzy ideal.
Proof: Let $A_i = \{(u, \mu_i^+(u), \mu_i^-(u), \lambda_i^+(u) | u \in U\}$ be a family of tripolar fuzzy ideals of CV-algebra U, then for any $u, v, w \in U$,

$(i)(\cap\mu_i^+)(u \star v) = inf(\mu_i^+(u \star v))$
$\geq inf(\mu_i^+(u))$
$= (\cap\mu_i^+)(u)$

$(ii)(\cap\mu_i^-)(u \star v) = inf(\mu_i^-(u \star v))$
$\leq inf(\mu_i^-(u))$
$= (\cap\mu_i^-)(u)$

$(iii)(\cap\lambda_i^+)(u \star v) = inf(\lambda_i^+(u \star v))$
$\leq inf(\lambda_i^+(u))$
$= (\cap\lambda_i^+)(u)$

$(iv)(\cap\mu_i^+)((u \star (v \star w)) \star w) = inf\ \mu_i^+(((u \star (v \star w)) \star w)$
$\geq inf(min\{\mu_i^+(u), \mu_i^+(v)\})$
$= min\{inf(\mu_i^+(u), inf(\mu_i^+(v))\}$
$= min\{(\cap\mu_i^+(u)), (\cap\mu_i^+(v))\}$

$(v)(\cap\mu_i^-)((u \star (v \star w)) \star w) = inf\ \mu_i^-(((u \star (v \star w)) \star w)$
$\leq inf(max\{\mu_i^-(u), \mu_i^-(v)\})$
$= max\{inf(\mu_i^-(u), inf(\mu_i^-(v))\}$
$= max\{(\cap\mu_i^-(u)), (\cap\mu_i^-(v))\}$

$(vi)(\cap\lambda_i^+)((u \star (v \star w)) \star w) = inf\ \lambda_i^+(((u \star (v \star w)) \star w)$
$\leq inf(max\{\lambda_i^+(u), \lambda_i^+(v)\})$
$= max\{inf(\lambda_i^+(u), inf(\lambda_i^+(v))\}$
$= max\{(\cap\lambda_i^+(u)), (\cap\lambda_i^+(v))\}.$
Hence the intersection of any set of tripolar fuzzy ideals of CV-algebra U is also a tripolar fuzzy ideal.

Theorem 5.2. The union of any set of tripolar fuzzy ideals of CV-algebra U is also a tripolar fuzzy ideal.
Proof: Let $A_i = \{(u, \mu_i^+(u), \mu_i^-(u), \lambda_i^+(u) | u \in U\}$ be a family of tripolar fuzzy ideals of CV-algebra U, then for any $u, v, w \in U$,

$(i)(\cup\mu_i^+)(u \star v) = sup(\mu_i^+(u \star v))$
$\geq sup(\mu_i^+(u))$
$= (\cup\mu_i^+)(u)$

$(ii)(\cup\mu_i^-)(u \star v) = sup(\mu_i^-(u \star v))$
$\leq sup(\mu_i^-(u))$
$= (\cup\mu_i^-)(u)$

$(iii)(\cup\lambda_i^+)(u\star v) = sup(\lambda_i^+(u\star v))$
$\leq sup(\lambda_i^+(u))$
$= (\cup\lambda_i^+)(u)$

$(iv)(\cup\mu_i^+)((u\star(v\star w))\star w) = sup\ \mu_i^+(((u\star(v\star w))\star w)$
$\geq sup(min\{\mu_i^+(u), \mu_i^+(v)\})$
$= min\{sup(\mu_i^+(u), sup(\mu_i^+(v))\}$
$= min\{(\cup\mu_i^+(u)), (\cup\mu_i^+(v))\}$

$(v)(\cup\mu_i^-)((u\star(v\star w))\star w) = sup\ \mu_i^-(((u\star(v\star w))\star w)$
$\leq sup(max\{\mu_i^-(u), \mu_i^-(v)\})$
$= max\{sup(\mu_i^-(u), sup(\mu_i^-(v))\}$
$= max\{(\cup\mu_i^-(u)), (\cup\mu_i^-(v))\}$

$(vi)(\cup\lambda_i^+)((u\star(v\star w))\star w) = sup\ \lambda_i^+(((u\star(v\star w))\star w)$
$\leq sup(max\{\lambda_i^+(u), \lambda_i^+(v)\})$
$= max\{sup(\lambda_i^+(u), sup(\lambda_i^+(v))\}$
$= max\{(\cup\lambda_i^+(u)), (\cup\lambda_i^+(v))\}$.
Hence the union of any set of tripolar fuzzy ideals of CV-algebra U is also a tripolar fuzzy ideal.

6|Product of Tripolar fuzzy ideal of CV-algebra

Definition 6.1. Let A and B be two tripolar fuzzy sets in U then the product $A\times B : U\times U \longrightarrow [0,1]$ is defined by $(A\times B)(u,v) = min\{A(u), B(v)\}$, for all $u, v\in U$.

Definition 6.2. Let $A = \{(u, \mu_1^+(u), \mu_1^-(u), \lambda_1^+(u))|u\in U\}$ and $B = \{(u, \mu_2^+(u), \mu_2^-(u), \lambda_2^+(u))|u\in U\}$ be two tripolar fuzzy sets of U, then the cartesian product $A\times B = \{\mu_1^+\times\mu_2^+, \mu_1^-\times\mu_2^-, \lambda_1^+\times\lambda_2^+\}$ is defined by

$$(\mu_1^+\times\mu_2^+)(u,v) = min\{\mu_1^+(u), \mu_2^+(v)\},\ (\mu_1^-\times\mu_2^-)(u,v) = max\{\mu_1^-(u), \mu_2^-(v)\} \text{ and } (\lambda_1^+\times\lambda_2^+)(u,v) = max\{\lambda_1^+(u), \lambda_2^+(v)\}.$$

Where $\mu_1^+\times\mu_2^+ : U\times U\longrightarrow [0,1]$, $\mu_1^-\times\mu_2^- : U\times U\longrightarrow[-1,0]$, and $\lambda_1^+\times\lambda_2^+ : U\times U\longrightarrow[0,1]$, for all $u, v\in U$.

Theorem 6.1. Let $A_1 = \{(u, \mu_1^+(u), \mu_1^-(u), \lambda_1^+(u)|u\in U_1\}$, $A_2 = \{(u, \mu_2^+(u), \mu_2^-(u), \lambda_2^+(u)|u\in U_2\}$, $\cdots$, $A_n = \{(u, \mu_n^+(u), \mu_n^-(u), \lambda_n^+(u)|u\in U_n\}$ be n- tripolar fuzzy sets of U, then $A_1\times A_2\times\cdots\times A_n$ is a tripolar fuzzy ideal of $U_1\times U_2\times\cdots\times U_n$.
Proof: Let $(u_1, u_2, ..., u_n), (v_1, v_2, ..., v_n), (w_1, w_2, ..., w_n)\in U_1\times U_2\times ...\times U_n$, we have

(i) $(\mu_1^+\times\mu_2^+\times ...\times\mu_n^+)((u_1, u_2, ..., u_n)\star(v_1, v_2, ..., v_n)) = \mu^+((u_1, u_2, ..., u_n)\star(v_1, v_2, ..., v_n))$
$= \mu^+((u_1\star v_1), (u_2\star v_2), ..., (u_n\star v_n))$
$= (\mu_1^+\times\mu_2^+\times ...\times\mu_n^+)((u_1\star v_1), (u_2\star v_2), ..., (u_n\star v_n))$
$\geq min\{\mu_1^+(u_1\star v_1), \mu_2^+(u_2\star v_2), ..., \mu_n^+(u_n\star v_n)\}$
$= min\{\mu_1^+(u_1), \mu_2^+(u_2), ..., \mu_n^+(u_n)\}$
$= (\mu_1^+\times\mu_2^+\times ...\times\mu_n^+)(u_1, u_2, ..., u_n)$,

(ii) $(\mu_1^-\times\mu_2^-\times ...\times\mu_n^-)((u_1, u_2, ..., u_n)\star(v_1, v_2, ..., v_n)) = \mu^-((u_1, u_2, ..., u_n)\star(v_1, v_2, ..., v_n))$
$= \mu^-((u_1\star v_1), (u_2\star v_2), ..., (u_n\star v_n))$
$= (\mu_1^-\times\mu_2^-\times ...\times\mu_n^-)((u_1\star v_1), (u_2\star v_2), ..., (u_n\star v_n))$
$\leq max\{\mu_1^-(u_1\star v_1), \mu_2^-(u_2\star v_2), ..., \mu_n^-(u_n\star v_n)\}$
$= max\{\mu_1^-(u_1), \mu_2^-(u_2), ..., \mu_n^-(u_n)\}$
$= (\mu_1^-\times\mu_2^-\times ...\times\mu_n^-)(u_1, u_2, ..., u_n)$,

(iii) $(\lambda_1^+\times\lambda_2^+\times ...\times\lambda_n^+)((u_1, u_2, ..., u_n)\star(v_1, v_2, ..., v_n)) = \lambda^+((u_1, u_2, ..., u_n)\star(v_1, v_2, ..., v_n))$
$= \lambda^+((u_1\star v_1), (u_2\star v_2), ..., (u_n\star v_n))$
$= (\lambda_1^+\times\lambda_2^+\times ...\times\lambda_n^+)((u_1\star v_1), (u_2\star v_2), ..., (u_n\star v_n))$
$\leq max\{\lambda_1^+(u_1\star v_1), \lambda_2^+(u_2\star v_2), ..., \lambda_n^+(u_n\star v_n)\}$

$= max\{\lambda_1^+(u_1), \lambda_2^+(u_2), ..., \lambda_n^+(u_n)\}$
$= (\lambda_1^+ \times \lambda_2^+ \times ...\lambda_n^+)(u_1, u_2, ..., u_n),$

(iv) $(\mu_1^+ \times \mu_2^+ \times ... \times \mu_n^+)(((u_1, u_2, ..., u_n) \star ((v_1, v_2, ..., v_n) \star (w_1, w_2, ..., w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^+(((u_1, u_2, ..., u_n) \star ((v_1, v_2, ..., v_n) \star (w_1, w_2, ..., w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^+(((u_1, u_2, ..., u_n) \star ((v_1 \star w_1), (v_2 \star w_2), ..., (v_n \star w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^+(((u_1 \star (v_1 \star w_1)), (u_2 \star (v_2 \star w_2)), ..., (u_n \star (v_n \star w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^+(((u_1 \star (v_1 \star w_1)) \star w_1), ((u_2 \star (v_2 \star w_2)) \star w_2), ..., ((u_n \star (v_n \star w_n)) \star w_n))$
$= (\mu_1^+ \times \mu_2^+ \times ... \times \mu_n^+)(((u_1 \star (v_1 \star w_1)) \star w_1), ((u_2 \star (v_2 \star w_2)) \star w_2), ..., ((u_n \star (v_n \star w_n)) \star w_1))$
$= min\{\mu_1^+((u_1 \star (v_1 \star w_1)) \star w_1), \mu_2^+((u_2 \star (v_2 \star w_2)) \star w_2), ..., \mu_n^+((u_n \star (v_n \star w_n)) \star w_n)\}$
$\geq min\{min\{\mu_1^+(u_1), \mu_1^+(v_1)\}, min\{\mu_2^+(u_2), \mu_2^+(v_2)\}, ..., min\{\mu_n^+(u_n), \mu_n^+(v_n)\}\}$
$= min\{min\{\mu_1^+(u_1), \mu_2^+(u_2), ..., \mu_n^+(u_n)\}, min\{\mu_1^+(v_1), \mu_2^+(v_2), ..., \mu_n^+(v_n)\}\}$
$=min\{(\mu_1^+ \times \mu_2^+ \times ... \times \mu_n^+)(u_1, u_2, ..., u_n), (\mu_1^+ \times \mu_2^+ \times ... \times \mu_n^+)(v_1, v_2, ..., v_n)\},$

(v) $(\mu_1^- \times \mu_2^- \times ... \times \mu_n^-)(((u_1, u_2, ..., u_n) \star ((v_1, v_2, ..., v_n) \star (w_1, w_2, ..., w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^-(((u_1, u_2, ..., u_n) \star ((v_1, v_2, ..., v_n) \star (w_1, w_2, ..., w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^-(((u_1, u_2, ..., u_n) \star ((v_1 \star w_1), (v_2 \star w_2), ..., (v_n \star w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^-(((u_1 \star (v_1 \star w_1)), (u_2 \star (v_2 \star w_2)), ..., (u_n \star (v_n \star w_n))) \star (w_1, w_2, ..., w_n))$
$= \mu^-(((u_1 \star (v_1 \star w_1)) \star w_1), ((u_2 \star (v_2 \star w_2)) \star w_2), ..., ((u_n \star (v_n \star w_n)) \star w_n))$
$= (\mu_1^- \times \mu_2^- \times ... \times \mu_n^-)(((u_1 \star (v_1 \star w_1)) \star w_1), ((u_2 \star (v_2 \star w_2)) \star w_2), ..., ((u_n \star (v_n \star w_n)) \star w_n))$
$= min\{\mu_1^-(((u_1 \star (v_1 \star w_1)) \star w_1), \mu_2^-((u_2 \star (v_2 \star w_2), ..., \mu_n^-(((u_n \star (v_n \star w_n)) \star w_n)) \star w_n))\}$
$\leq min\{max\{\mu_1^-(u_1), \mu_1^-(v_1)\}, max\{\mu_2^-(u_2), \mu_2^-(v_2)\}, ..., max\{\mu_n^-(u_n), \mu_n^-(v_n)\}\}$
$= min\{max\{\mu_1^-(u_1), \mu_2^-(u_2), ..., \mu_n^-(u_n)\}, max\{\mu_1^-(v_1), \mu_2^-(v_2), ..., \mu_n^-(v_n)\}\}$
$= max\{min\{\mu_1^-(u_1), \mu_2^-(u_2), ..., \mu_n^-(u_n)\}, min\{\mu_1^-(v_1), \mu_2^-(v_2), ..., \mu_n^-(v_n)\}\}$
$=max\{(\mu_1^- \times \mu_2^- \times ... \times \mu_n^-)(u_1, u_2, ..., u_n), (\mu_1^- \times \mu_2^- times... \times \mu_n^-)(v_1, v_2, ..., v_n)\}$

(vi) $(\lambda_1^+ \times \lambda_2^+ \times ... \times \lambda_n^+)(((u_1, u_2, ..., u_n) \star ((v_1, v_2, ..., v_n) \star (w_1, w_2, ..., w_n))) \star (w_1, w_2, ..., w_n))$
$= \lambda^+(((u_1, u_2, ..., u_n) \star ((v_1, v_2, ..., v_n) \star (w_1, w_2, ..., w_n))) \star (w_1, w_2, ..., w_n))$
$= \lambda^+(((u_1, u_2, ..., u_n) \star ((v_1 \star w_1), (v_2 \star w_2), ..., (v_n \star w_n))) \star (w_1, w_2, ..., w_n))$
$= \lambda^+(((u_1 \star (v_1 \star w_1)), (u_2 \star (v_2 \star w_2)), ..., ((u_n \star (v_n \star w_n))) \star (w_1, w_2, ..., w_n))$
$= \lambda^+(((u_1 \star (v_1 \star w_1)) \star w_1), ((u_2 \star (v_2 \star w_2)) \star w_2), ..., ((u_n \star (v_n \star w_n)) \star w_n))$
$= (\lambda_1^+ \times \lambda_2^+ \times ... \times \lambda_n^+)(((u_1 \star (v_1 \star w_1)) \star w_1), ((u_2 \star (v_2 \star w_2)) \star w_2), ..., ((u_n \star (v_n \star w_n)) \star w_n))$
$= min\{\lambda_1^+(((u_1 \star (v_1 \star w_1)) \star w_1), \lambda_2^+((u_2 \star (v_2 \star w_2)) \star w_2), ..., \lambda_n^+(((u_n \star (v_n \star w_n)) \star w_n))\}$
$\leq min\{max\{\lambda_1^+(u_1), \lambda_1^+(v_1)\}, max\{\lambda_2^+(u_2), \lambda_2^+(v_2)\}, ..., max\{\lambda_n^+(u_n), \lambda_n^+(v_n)\}\}$
$= min\{max\{\lambda_1^+(u_1), \lambda_2^+(u_2), ..., \lambda_n^+(u_n)\}, max\{\lambda_1^+(v_1), \lambda_2^+(v_2), ..., \lambda_n^+(v_n)\}\}$
$= max\{min\{\lambda_1^+(u_1), \lambda_2^+(u_2), ..., \lambda_n^+(u_n)\}, min\{\lambda_1^+(v_1), \lambda_2^+(v_2), ..., \lambda_n^+(v_n)\}\}$
$=max\{(\mu_1^+ \times \mu_2^+ \times ... \times \mu_n^+)(u_1, u_2, ..., u_n), (\mu_1^+ \times \mu_2^+ \times ... \times \mu_n^+)(v_1, v_2, ..., v_n)\}.$
Hence $A_1 \times A_2 \times \cdots \times A_n$ is a tripolar fuzzy ideal of $U_1 \times U_2 \times \cdots \times U_n$.

7|Homomorphism of Tripolar fuzzy ideal of CV-algebra

Definition 7.1. Let $(U, \star, 0)$ and $(V, \star', 0')$ be CV-algebras. A mapping $\varphi : U \longrightarrow V$ is said to be a homomorphism if $\varphi(u \star v) = \varphi(u) \star' \varphi(v)$ for all $u, v \in U$. Note that $\varphi(0) = 0$.

Definition 7.2. Let $\varphi : U \longrightarrow V$ be a homomorphism of CV-algebras for any tripolar fuzzy set $A = \{(u, \mu^+(u), \mu^-(u), \lambda^+(u))|u \in U\}$ in U and we define $A_\varphi = \{(u, \mu_\varphi^+(u), \mu_\varphi^-(u), \lambda_\varphi^+(u))|u \in U\}$ in V by $\mu_\varphi^+(u) = \mu^+(\varphi(u))$, $\mu_\varphi^-(u) = \mu^-(\varphi(u))$ and $\lambda_\varphi^+(u) = \lambda^+(\varphi(u))$, for all $u \in U$.

Theorem 7.1. Let $\varphi : U \longrightarrow V$ be a homomorphism of CV-algebras. If $A = \{(u, \mu^+(u), \mu^-(u), \lambda^+(u))|u \in U\}$ is a tripolar fuzzy ideal of U, then $A_\varphi = \{(u, \mu_\varphi^+(u), \mu_\varphi^-(u), \lambda_\varphi^+(u))|u \in U\}$ is a tripolar fuzzy ideal of V.
Proof: Let $u, v, w \in U$ and A is a tripolar fuzzy ideal of U, we have

(i) $\mu_\varphi^+(u \star v) = \mu^+(\varphi(u \star v))$
$= \mu^+(\varphi(u) \star \varphi(v))$

$\geq \mu^+(\varphi(u))$
$= \mu^+_\varphi(u)$

(ii) $\mu^-_\varphi(u \star v) = \mu^-(\varphi(u \star v))$
$= \mu^-(\varphi(u) \star \varphi(v))$
$\leq \mu^-(\varphi(u)) = \mu^-_\varphi(u)$

(iii) $\lambda^+_\varphi(u \star v) = \lambda^+(\varphi(u \star v))$
$= \lambda^+(\varphi(u) \star \varphi(v))$
$\leq \lambda^+(\varphi(u))$
$= \lambda^+_\varphi(u)$

(iv) $\mu^+_\varphi((u \star (v \star w)) \star w) = \mu^+(\varphi(((u \star (v \star w)) \star w)))$
$= \mu^+((\varphi((u \star (v \star w))) \star \varphi(w))$
$= \mu^+((\varphi(u) \star (\varphi(v \star w))) \star \varphi(w))$
$= \mu^+((\varphi(u) \star (\varphi(v) \star \varphi(w))) \star \varphi(w))$
$\geq min\{\mu^+(\varphi(u)), \mu^+(\varphi(v))\}$
$= min\{\mu^+_\varphi(u), \mu^+_\varphi(v)\}$

(v) $\mu^-_\varphi((u \star (v \star w)) \star w) = \mu^-(\varphi(((u \star (v \star w)) \star w)))$
$= \mu^-((\varphi((u \star (v \star w))) \star \varphi(w))$
$= \mu^-((\varphi(u) \star (\varphi(v \star w))) \star \varphi(w))$
$= \mu^-((\varphi(u) \star (\varphi(v) \star \varphi(w))) \star \varphi(w))$
$\leq max\{\mu^-(\varphi(u)), \mu^-(\varphi(v))\}$
$= max\{\mu^-_\varphi(u), \mu^-_\varphi(v)\}$

(vi) $\lambda^+_\varphi((u \star (v \star w)) \star w) = \lambda^+(\varphi(((u \star (v \star w)) \star w)))$
$= \lambda^+((\varphi((u \star (v \star w))) \star \varphi(w))$
$= \lambda^+((\varphi(u) \star (\varphi(v \star w))) \star \varphi(w))$
$= \lambda^+((\varphi(u) \star (\varphi(v) \star \varphi(w))) \star \varphi(w))$
$\leq max\{\lambda^+(\varphi(u)), \lambda^+(\varphi(v))\}$
$= max\{\lambda^+_\varphi(u), \lambda^+_\varphi(v)\}$.
Hence A_φ is a tripolar fuzzy ideal of V.

Theorem 7.2. Let $\varphi : U \longrightarrow V$ be an epimorphism of CV-algebras. If $A_\varphi = \{(u, \mu^+_\varphi(u), \mu^-_\varphi(u), \lambda^+_\varphi(u)) | u \in U\}$ is a tripolar fuzzy ideal of V, then $A = \{(u, \mu^+(u),$
$\mu^-(u), \lambda^+(u)) | u \in U\}$ is a tripolar fuzzy ideal of U.
Proof: For any $x, y, z \in V$, there exists $u, v, w \in U$ such that $\varphi(x) = u$, $\varphi(y) = v$ and $\varphi(z) = w$.

(i) $\mu^+(u \star v) = \mu^+(\varphi(x) \star \varphi(y))$
$= \mu^+(\varphi(x \star y))$
$= \mu^+_\varphi(x \star y)$
$\geq \mu^+_\varphi(x)$
$= \mu^+(\varphi(x))$
$= \mu^+(u)$

(ii) $\mu^-(u \star v) = \mu^-(\varphi(x) \star \varphi(y))$
$= \mu^-(\varphi(x \star y))$
$= \mu^-_\varphi(x \star y)$
$\leq \mu^-_\varphi(x)$
$= \mu^-(\varphi(x)) = \mu^-(u)$

(iii) $\lambda^+(u \star v) = \lambda^+(\varphi(x) \star \varphi(y))$
$= \lambda^+(\varphi(x \star y))$
$= \lambda^+_\varphi(x \star y)$
$\leq \lambda^+_\varphi(x)$

$= \lambda^+(\varphi(x))$
$= \lambda^+(u)$

(iv) $\mu^+((u \star (v \star w)) \star w) = \mu^+((\varphi(x) \star (\varphi(y) \star \varphi(z))) \star \varphi(z))$
$= \mu^+((\varphi(x) \star (\varphi(y \star z))) \star \varphi(z))$
$= \mu^+(\varphi(x \star (y \star z)) \star \varphi(z))$
$= \mu^+(\varphi(x \star (y \star z)) \star z))$
$= \mu^+_\varphi((x \star (y \star z)) \star z)$
$\geq min\{\mu^+_\varphi(x), \mu^+_\varphi(y)\}$
$= min\{\mu^+(\varphi(x)), \mu^+(\varphi(y))\}$
$= min\{\mu^+(u), \mu^+(v)\}$

(v) $\mu^-((u \star (v \star w)) \star w) = \mu^-((\varphi(x) \star (\varphi(y) \star \varphi(z))) \star \varphi(z))$
$= \mu^-((\varphi(x) \star (\varphi(y \star z))) \star \varphi(z))$
$= \mu^-(\varphi(x \star (y \star z)) \star \varphi(z))$
$= \mu^-(\varphi(x \star (y \star z)) \star z))$
$= \mu^-_\varphi((x \star (y \star z)) \star z)$
$\leq max\{\mu^-_\varphi(x), \mu^-_\varphi(y)\}$
$= max\{\mu^-(\varphi(x)), \mu^-(\varphi(y))\}$
$= max\{\mu^-(u), \mu^-(v)\}$

(vi) $\lambda^+((u \star (v \star w)) \star w) = \lambda^+((\varphi(x) \star (\varphi(y) \star \varphi(z))) \star \varphi(z))$
$= \lambda^+((\varphi(x) \star (\varphi(y \star z))) \star \varphi(z))$
$= \lambda^+(\varphi(x \star (y \star z)) \star \varphi(z))$
$= \lambda^+(\varphi(x \star (y \star z)) \star z))$
$= \lambda^+_\varphi((x \star (y \star z)) \star z)$
$\leq max\{\lambda^+_\varphi(x), \lambda^+_\varphi(y)\}$
$= max\{\lambda^+(\varphi(x)), \lambda^+(\varphi(y))\}$
$= max\{\lambda^+(u), \lambda^+(v)\}$.
Hence A is a tripolar fuzzy ideal of U.

8|Application of Tripolar fuzzy ideal of CV-algebra

Let U be a set consisting of five students u, v, w, x, y ie., $U = \{u, v, w, x, y\}$. They have a result consisting of three aspects, the result of u is $A(u) =< 0.76, 0.82, 0.52 >$, where 0.5 represents *ok* or *notbad*. Suppose $A(v) =< 0.86, 0.81, 0.54 >$, $A(w) =< 0.68, 0.72, 0.51 >$, $A(x) =< 0.79, 0.83, 0.57 >$, $A(y) =< 0.64, 0.89, 0.50 >$. Then we obtain tripolar fuzzy ideal of CV-algebra by the Cayley table,

$\star$	u	v	w	x	y
μ^+	*0.76*	*0.86*	*0.68*	*0.79*	*0.64*
μ^-	*0.82*	*0.81*	*0.72*	*0.83*	*0.89*
λ^+	*0.52*	*0.54*	*0.0.51*	*0.57*	*0.50*

9|Conclusion

In this paper we have introduced CV-algebra. We have proved some of its properties like self-distributiveness and essence. We have given the condition for an existence of ideals, fuzzy ideals and tripolar fuzzy ideals in CV-algebra and we have proved some of its results. We have defined the product and homomorphism of tripolar fuzzy ideal of CV-algebra.

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