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An In-Depth Analysis of Restricted and Extended Lambda Operations for Soft Sets

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Abstract

Since its introduction by Molodtsov in 1999, soft set theory has gained widespread recognition as a method for modeling uncertainty and handling problems involving uncertainty. It has been used in several theoretical and practical situations. Since the theory's inception, scholars have been intrigued by its central idea-soft set operations. Several extended and restricted operations were defined, and their properties were studied. We provide new restricted and extended soft set operations that we call restricted lambda and extended lambda operations and examine their basic algebraic properties in depth. The distributions of this operation over other soft-set operations are also investigated. We demonstrate that the extended lambda operation, when combined with other kinds of soft sets, forms several significant algebraic structures, such as semirings and nearsemirings in the collection of soft sets over the universe, by taking into account the algebraic properties of the operation and its distribution rules. This theoretical research is very important both theoretically and practically, as the primary idea of the theory is the operations of soft sets, as they serve as the foundation for numerous applications, including cryptology, as well as the decision-making processes.

Keywords: Soft sets, Soft set operations, Restricted lambda operation, Extended lambda operation.

1 | Introduction

In the real world, there is a lot of uncertainty. To handle these ambiguities, traditional mathematical reasoning is inadequate. More scientific investigation beyond the reach of currently accessible methods has been

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necessary to dispel these uncertainties. In this respect, when Pascal and Fermat analyzed the uncertainty problem analytically in the early 17th century, they presented the probability theory. In the early 19th century, a large number of scientists investigated uncertainty.

Heisenberg first explained uncertainty in 1920 and opened the door to many values. Early in the 1930s, Lukaisewicz developed the first three-valued logic system. Some theories that may be used to describe uncertainty include probability theory, interval mathematics, and fuzzy set theory; however, each has drawbacks. Consequently, Molodtsov [1] presented the theory of Soft Set in 1999, independent of creating the membership function. Unlike fuzzy set theory, which aims to eliminate uncertainty, soft set theory employs a set-valued function instead of a real-valued one. This idea has been successfully applied to several mathematical fields since its introduction. The domains are measurement theory, game theory, probability theory, Riemann integration, and Perron integration analysis.

Maji et al. [2] and Pei and Miao [3] conducted the first study on soft-set operations. Ali et al. [4] presented many soft set operations, including restricted and extended soft set operations. In their work on soft sets, Sezgin and Yavuz [5] studied on soft binary piecewise symmetric difference operation of soft sets. A thorough examination of the algebraic structures of soft sets was carried out by Ali et al. [6]. A number of academics were interested in soft set operations and studied the subject matter in depth in [7–16].

Several novel forms of soft-set operations have been introduced in the last five years. Eren and Çalışıcı [17] examined the idea and characteristics of the soft binary piecewise difference operation in soft sets. Sezgin and Çağman [18] developed the complementary soft binary piecewise difference operations of soft sets, and Stojanovic [19] defined and investigated its characteristics. Furthermore, a comprehensive the complementary soft binary piecewise theta operation was conducted by Sezgin and Sarıalioğlu [20].

Sezgin et al. [21] worked on many new binary set operations and outlined several more, motivated by the work of Çağman [22], who added two new complement operations to the literature. Aybek [23] proposed several novel restricted and extended soft set operations using the method. Complementary extended soft set operations were the focus of Akbulut [24], Demirci [25], and Sarıalioğlu [26] in their attempts to modify the structure of extended operations in soft sets.

Classifying algebraic structures and finding, displaying, and deriving results from their common properties are the goals of abstract algebra. This is why abstract algebra is given to this area of mathematics. Mathematicians have studied algebraic structures for millennia as they offer a universal and abstract approach to understanding and comprehending mathematical subjects. Fundamentally, algebraic structures are involved in many branches of mathematics. There are several significant uses for algebraic structures like rings, groups, and fields in mathematics, as well as other domains like physics and computer science. A foundation for comprehending more complex mathematical concepts and structures is laid by the structures of algebraic geometry (the study of multivariable polynomial solutions), algebraic topology, modular arithmetic, physics, number theory, and computer graphics, among other highly relevant fields.

Furthermore, a framework for analyzing and comprehending a variety of mathematical objects and their relationships is provided by mathematical structures. Particular groups have applications in physics, chemistry, and cryptography and are used to analyze symmetries, rotations, and transformations in mathematical contexts. Studying the symmetries of intriguing geometric objects and forms requires using fundamental groups and their representations as group transformations, which are fundamental algebraic structures. Number theory, coding theory, and abstract algebra all use rings. Fundamental to geometry and other branches of mathematics is field algebra. Engineering, quantum physics, and linear algebra all use vector spaces. Algebras are used in computer science, physics, and mathematical reasoning. Both representation theory and abstract algebra make use of modules.

Moreover, abstract algebra, which examines many algebraic systems' shared structures and common features, depends heavily on studying algebraic structures. With a knowledge of these structures' features, mathematicians can solve intricate problems, create new theories, and apply ideas to a variety of mathematical, scientific, and technical domains. Furthermore, applications frequently provide special examples of algebraic structures, which help to clarify specific circumstances and make it easier to examine more general scenarios.

Classifying algebraic structures according to the properties of the operation given on a set is one of the most important algebraic mathematics problems. We might suggest new soft set operations, examine their properties, and think about the algebraic structures they form in the collection of soft sets to further our grasp of this subject. Thus far, four extended soft set operations (extended intersection, union, difference, and symmetric difference for soft sets) and four restricted soft set operations (restricted intersection, union, difference, and symmetric difference) have been developed. With the aim of making a major contribution to the field of soft set theory, we refer to the new restricted and extended soft set operations as restricted lambda operation and extended lambda operations of soft sets, which we propose in this study, and closely examine the algebraic structures associated with them as well as other soft set operations in the collection of soft sets. This study is organized as follows. the basic ideas behind soft sets and other algebraic structures are reviewed in Section 2. And in Section 3, the new soft set operations are defined. A thorough analysis is conducted on the algebraic characteristics of the first restricted lambda soft set operation and the second extended lambda soft set operation. We also study the distribution rules of these operations over other types of soft-set operations. Considering the distribution laws and the algebraic properties of the soft set operations, a detailed analysis of the algebraic structures formed in the set of soft sets with these operations is provided.

We demonstrate that i the universe's collection of soft sets, a number of significant algebraic structures, including semiring and seminearring, are formed. A comprehensive analysis improves our knowledge of the applications and implications of soft set theory in many different fields. In the conclusion section, we discuss the significance of the study's findings and potential applications.

2 | Preliminaries

This section covers a number of algebraic structures as well as a number of basic concepts in soft set theory.

Definition 1 ([27]). Let U be the universal set, E be the parameter set, P(U) be the power set of U, and $T\subseteq E$. A pair (F, T) is called a soft set on U. Here, F is a function given by $F: T \rightarrow P(U)$.

Throughout this paper, the collection of all the soft sets over U is designated by $S_E(U)$ and $S_T(U)$ denotes the collection of all soft sets over U with a fixed parameter set T, where T is a subset of E.

Definition 2 ([6]). Let (F,T) be a soft set over U. If $F(x)=\emptyset$ for all $x\in T$, then the soft set (F,T) is called a null soft set with respect to K, denoted by $\emptyset_K$. If $F(x)=\emptyset$ for all $x\in E$, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ [4]. A soft set with an empty parameter set is denoted as $\emptyset_\emptyset$. It is obvious that $\emptyset_\emptyset$ is the only soft set with an empty parameter set.

Definition 3 ([4]). Let (F,T) be a soft set over U. If $F(x)=U$ for all $x\in T$, then the soft set (F,T) is called a relative whole soft set with respect to T, denoted by U_T. If $F(x)=U$ for all $x\in E$, then the soft set (F,E) is called an absolute soft set and denoted by U_E.

Definition 4 ([3]). Let (F,T) and (G,Y) be soft sets over U. If $T\subseteq Y$ and for all $x\in T$, $F(x)\subseteq G(x)$, then (F,T) is said to be a soft subset of (G,Y), denoted by $(F,T)\tilde{\subseteq}(G,Y)$. If $(F,T)\tilde{\subseteq}(G,Y)$ and $(G,Y)\tilde{\subseteq}(F,T)$, then (F,T) and (G,Y) are called soft equal sets.

Definition 5 ([4]). Let (F,T) be a soft set over U. The relative complement of (F,T), denoted by $(F,T)^r=(F^r,T)$, is defined as follows: $F^r(x)=U-F(x)$ for all $x\in T$.

For two sets X and Y, $X+Y=X'\cup Y$ and $X\theta Y=X'\cap Y'$, $X*Y=X'\cup Y'$, $X\gamma Y= X'\cap Y$, $X\lambda Y=X\cup Y'$. Let "$\ominus$" be used to represent the set operations.

Definition 6 ([4], [23]). Let (F, T) and (G, Y) be two soft sets over U. The restricted $\ominus$ operation of (F, T) and (G, Y) is the soft set (H, Z), denoted by $(F,T)\ominus_R(G,Y)=(H,Z)$, where $Z=T\cap Y\neq\emptyset$ and for all $x\in Z$, $H(x)=F(x)\ominus G(x)$. Here, if $Z=T\cap Y=\emptyset$, then $(F,T)\ominus_R(G,Y)=\emptyset_\emptyset$.

Definition 7 ([2], [4]). Let (F, T) and (G, Y) be two soft sets over U. The extended ⊜ operation (F, T) and (G,Y) is the soft set (H,Z), denoted by $(F, T) \ominus_\varepsilon (G, Y) = (H, Z)$, where Z=T∪Y, and for all x∈Z.

$$H(x) = \begin{cases} F(x). & x \in T - Y, \\ G(x). & x \in Y - T, \\ F(x) \ominus G(x). & x \in T \cap Y, \end{cases}$$

Definition 8 ([24–26]). Let (F,T) and (G, Y) be two soft sets over U. The complementary extended $\ominus_\varepsilon$ operation (F,T) and (G,Y) is the soft set (H,Z), denoted by $(F.T) \overset{*}{\ominus}_\varepsilon (G, Y) = (H, Z)$, where Z=T∪Y, and for all x∈Z.

$$H(x) = \begin{cases} F'(x). & x \in T - Y, \\ G'(x). & x \in Y - T, \\ F(x) \ominus G(x). & x \in T \cap Y, \end{cases}$$

Definition 9 ([5], [28]). Let (F,T) and (G,Y) be two soft sets on U. The soft binary piecewise ⊜ operation of (F,T) and (G,Y) is the soft set (H,T), denoted by $(F.T) \overset{\sim}{\ominus} (G.Y) = (H.T)$, where for all x∈T.

$$H(x) = \begin{cases} F(x). & x \in T - Y, \\ F(x) \ominus G(x). & x \in T \cap Y, \end{cases}$$

Definition 10 ([18], [20]). Let (F,T) and (G,Y) be two soft sets on U. The complementary soft binary piecewise ⊜ operation of (F,T) and (G,Y) is the soft set (H,T), denoted by $(F, T) \underset{\ominus}{\overset{*}{\sim}} (G, Y) = (H, T)$, where for all x∈T

$$H(x) = \begin{cases} F'(x). & x \in T - Y, \\ F(x) \bowtie G(x). & x \in T \cap Y, \end{cases}$$

For more about soft sets, we refer to [29–54].

For more on the band, (bounded) semilattice bounded semilattice, we refer to [55]; for more on semiring and hemiring, we refer to [56]; and for more on nearsemiring (or seminearring), we refer to [56]. Regarding the prospective effects of network analysis and graph applications on soft, sets-which are assigned by the divisibility of determinants, we refer to [57].

3 | Restricted and Extended Lambda Operation

This section introduces a new restricted and extended soft set operation called the restricted lambda and extended lambda. It examines the distributive rules over other types of soft sets, their relationship with other soft set operations, and their algebraic properties. It also investigated which algebraic structures these operations form on the $S_E(U)$ set, leading to important results.

3.1 | Restricted Lambda Operation and its Properties

Definition 11. Let (F, T) and (G, Z) be soft sets over U, The restricted lambda of (F. T) and (G. Z), denoted by $(F.T)\lambda_R(G.Z)$. is defined as $(F.T)\lambda_R(G.Z) = (H.C)$, where C=T∩Z, and if C=T∩Z≠∅, then for all α∈C, H(α)=F(α)λG(α)=F(α)∪G'(α); if C=T∩Z=∅, then $(F,T)\lambda_R(G,Z)=(H, C)= \emptyset_\emptyset$.

Since the only soft set with an empty parameter set is $\emptyset_\emptyset$, if C=T∩Z= ∅, then it is obvious that $(F,T)\lambda_R(G,Z)= \emptyset_\emptyset$. Thus, in order to define the restricted lambda operation of (F. T) and (G. Z), there is no condition that T∩Z≠ ∅.

Example 1. Let $E=\{e_1,e_2,e_3,e_4\}$be the parameter set, $T = \{e_1.e_3\}$ and $Z = \{e_2.e_3.e_4\}$ be subsets of E, $U = \{h_1.h_2.h_3.h_4.h_5\}$ be the universal set, (F,T) and (G,Z) be the soft sets over U as $(F,T) = \{(e_1,.\{h_2.h_5),$ $(e_3,\{h_1,h_2,h_5\})\}$, $(G,Z)=\{(e_2,\{h_1,h_4,h_5\}),$ $\{(e_3,\{h_2,h_3,h_4\}),$ $(e_4.h_3,h_5\})\}$. Here, let $(F,T)\lambda_R(G,Z)=(H,T\cap Z),$

where for all $\alpha \in T \cap Z=\{e_3\}$. Thus, $H(\alpha)=F(\alpha) \cup G'(\alpha), H(e_3)=F(e_3) \cup G'(e_3)=\{h_1,h_2,h_5\} \cup \{h_1,h_5\}=\{h_1,h_2,h_5\}$. Thus,

$$(F,T)\lambda_R(G,Z)=\{(e_3,\{h_1,h_2,h_5\})\}.$$

Theorem 1. Algebraic properties of the operation: Let (F,T), (G,T), (H,T), (G,Z), (H,M), (K,V) and (L,V) be soft sets over U. Then,

I. The set $S_E(U)$ in closed under λ_R.

Proof: it is clear that λ_R is a binary operation in $S_E(U)$. That is

$$\lambda_R: S_E(U) x\, S_E(U) \rightarrow S_E(U).$$

$$((F,T),(G,Z)) \rightarrow (F,T)\lambda_R(G,Z)=(H,T\cap Z).$$

similarly,

$$\lambda_R: S_T(U) x\, S_T(U) \rightarrow S_T(U).$$

$$((F,T),(G,T)) \rightarrow (F,T)\lambda_R(G,T)=(H,T\cap T)=(H,T).$$

That is, let T be a fixed subset of the set E and (F,T) and (G,T) be elements of $S_T(U)$, then so is (F,T) λ_R(G,T). Namely, $S_T(U)$ is closed under λ_R either.

$$[(F,T)\lambda_R(G,Z)]\lambda_R(H,M) \neq (F,T)\lambda_R[(G,Z)\lambda_R(H,M)].$$

Proof: let $(F.T)\lambda_R(G.Z)=(S.T\cap Z)$, where for all $\alpha \in T \cap Z$, $T(\alpha)=F(\alpha)\cup G'(\alpha)$. Let $(S,T\cap Z)\lambda_R(H,M)=(R,(T\cap Z)\cap M))$, where for all $\alpha \in (T\cap Z)\cap M$, $R(\alpha)=T(\alpha)\cup H'(\alpha)$. Thus,

$$R(\alpha)=[F(\alpha)\cup G'(\alpha)]\cup H'(\alpha).$$

Let $(G.Z)\lambda_R(H.M)=(K.Z\cap M)$, where for all $\alpha \in Z\cap M$, $K(\alpha)=G(\alpha)\cup H'(\alpha)$. Let $(F,T)\lambda_R(K.Z\cap M)=(S,T\cap(Z\cap M))$, where for all $\alpha \in T\cap(Z\cap M)$, $S(\alpha)=F(\alpha)\cup K'(\alpha)$. Thus,

$$S(\alpha)=F(\alpha)\cup[G'(\alpha)\cap H(\alpha)].$$

Thus, $(R,(T\cap Z)\cap M) \neq (S,T\cap(Z\cap M))$. That is, in S_E(U), the operation λ_R is not associative. Here, it is obvious that if $T\cap Z=\emptyset$ or $Z\cap M=\emptyset$ or $T\cap M=\emptyset$, then since both sides of the equality are $\emptyset_\emptyset$, the operation λ_R is associative under these conditions.

$$[(F,T)\lambda_R(G,T)]\lambda_R(H,T) \neq (F,T)\lambda_R[(G,T)\lambda_R(H,T)].$$

Proof: let $(F,T)\lambda_R(G,T)=(K,T)$, where for all $\alpha \in T\cap T=T$, $K(\alpha)=F(\alpha)\cup G'(\alpha)$. Let $(K,T)\lambda_R(H,T)=(R,T)$, where for all $\alpha \in T\cap T=T$, $R(\alpha)=K(\alpha)\cup H'(\alpha)$. Hence,

$$R(\alpha)=[F(\alpha)\cup G'(\alpha)]\cup H'(\alpha).$$

Let $(G,T)\lambda_R(H,T)=(L,T)$, where for all $\alpha \in T\cap T$, $L(\alpha)=G(\alpha)\cup H'(\alpha)$. Let $(F,T)\lambda_R(L,T)=(N,T)$, where for all $\alpha \in T\cap T$, $N(\alpha)=F(\alpha)\cup L'(\alpha)$. Hence,

$$N(\alpha)=F(\alpha)\cup[G'(\alpha)\cap H(\alpha)].$$

Thus, $(R,T)\neq(N,T)$. That is, λ_R is not associative in the collection of soft sets with a fixed parameter set.

$$(F,T)\lambda_R(G,Z) \neq (G,Z)\lambda_R(F,T).$$

Proof: let $(F,T)\lambda_R(G,Z)=(H,T\cap Z)$, where for all $\alpha \in T\cap Z$, $H(\alpha)=F(\alpha)\cup G'(\alpha)$. Let $(G,Z)\lambda_R(F,T)=(S,Z\cap T)$, where for all $\alpha \in Z\cap T$, $S(\alpha)=G(\alpha)\cup F'(\alpha)$. Thus,

$$(F,T)\lambda_R(G,Z) \neq (G,Z)\lambda_R(F,T).$$

That is, λ_R is not commutative in $S_E(U)$. Here, it is obvious that if $T \cap Z = \emptyset$, then since both sides are $\emptyset_\emptyset$, λ_R is commutative in $S_E(U)$ under this condition. Moreover, it is evident that $(F,T)\lambda_R(G,T) \neq (G,T)\lambda_R(F,T)$, namely, λ_R is not commutative in the collection of soft sets with a fixed parameter set.

$(F,T)\lambda_R(F,T) = U_T$.

Proof: let $(F,T)\lambda_R(F,T) = (H, T \cap T)$. Thus, for all $\alpha \in T$, $H(\alpha) = F(\alpha) \cup F'(\alpha) = U$. Hence, $(H,T) = U_T$.

That is, the operation λ_R is not idempotent in $S_E(U)$.

$(F,T)\lambda_R \emptyset_T = U_T$.

Proof: let $\emptyset_T = (S,T)$, where for all $\alpha \in T$, $S(\alpha) = \emptyset$. Let $(F,T)\lambda_R(S,T) = (H, T \cap T)$, where for all $\alpha \in T$, $H(\alpha) = F(\alpha) \cup S'(\alpha) = F(\alpha) \cup U = U$. Thus, $(H,T) = U_T$.

$\emptyset_T \lambda_R (F,T) = (F,T)^r$,

Proof: let $\emptyset_T = (S,T)$, where for all $\alpha \in T$, $S(\alpha) = \emptyset$. Let $(S,T)\lambda_R(F,T) = (H, T \cap T)$, where for all $\alpha \in T$, $H(\alpha) = S(\alpha) \cup F'(\alpha) = \emptyset \cup F'(\alpha) = F'(\alpha)$. Thus, $(H,T) = (F,T)^r$.

$(F,T)\lambda_R \emptyset_M = U_{T \cap M}$.

Proof: let $\emptyset_M = (S,M)$, where for all $\alpha \in M$, $S(\alpha) = \emptyset$. Let $(F,T)\lambda_R(S,M) = (H, T \cap M)$, where for all $\alpha \in T \cap M$, $H(\alpha) = F(\alpha) \cup S'(\alpha) = F(\alpha) \cup U = U$. Thus, $(H, T \cap M) = U_{T \cap M}$.

$\emptyset_M \lambda_R (F,T) = (F, M \cap T)$.

Proof: let $\emptyset_M = (S,M)$, where for all $\alpha \in M$, $S(\alpha) = \emptyset$. Let $(S,M)\lambda_R(F,T) = (H, M \cap T)$, where for all $\alpha \in M \cap T$, $H(\alpha) = S(\alpha) \cup F'(\alpha) = \emptyset \cup F(\alpha) = F(\alpha)$. Thus, $(H, M \cap T) = (F, M \cap T)$.

$(F,T)\lambda_R \emptyset_E = U_T$.

Proof: let $\emptyset_E = (S,E)$, where for all $\alpha \in E$, $S(\alpha) = \emptyset$. Let $(F,T)\lambda_R(S,E) = (H, T \cap E)$, where for all $\alpha \in T \cap E = T$, $H(\alpha) = F(\alpha) \cup S'(\alpha) = F(\alpha) \cup U = U$. Thus, $(H,T) = U_T$.

$\emptyset_E \lambda_R (F,T) = (F,T)^r$.

Proof: let $\emptyset_E = (S,E)$, where for all $\alpha \in E$, $S(\alpha) = \emptyset$. Let $(S,E)\lambda_R(F,T) = (H, E \cap T)$, where for all $\alpha \in e \cap t = t$, $H(\alpha) = S(\alpha) \cup F'(\alpha) = \emptyset \cup F'(\alpha) = F'(\alpha)$. Thus, $(H,T) = (F,T)^r$.

$(F,T)\lambda_R \emptyset_\emptyset = \emptyset_\emptyset \lambda_R (F,T) = \emptyset_\emptyset$.

Proof: let $\emptyset_\emptyset = (S, \emptyset)$. Thus, $(F,T)\lambda_R(S,\emptyset) = (H, T \cap \emptyset) = (H, \emptyset)$. Since $\emptyset_\emptyset$ is the only soft set with the empty parameter set, $(H, \emptyset) = \emptyset_\emptyset$. That is, the absorbing element of λ_R in $S_E(U)$ is the soft set

$(F,T)\lambda_R U_T = (F,T)$.

Proof: let $U_T = (K,T)$, where for all $\alpha \in T$, $K(\alpha) = U$. Let $(F,T)\lambda_R(K,T) = (H, T \cap T)$, where for all $\alpha \in T$, $H(\alpha) = F(\alpha) \cup T'(\alpha) = F(\alpha) \cup \emptyset = (F,T)$. Thus, $(H,T) = (F,T)$. That is, the right identity element of λ_R in $S_T(U)$ is the soft set U_T.

$U_T \lambda_R (F,T) = U_T$.

Proof: let $U_T = (K.T)$, where for all $\alpha \in T. K(\alpha) = U$. Let $(K.T)\lambda_R(F.T) = (H. T \cap T)$, where for all $\alpha \in T$, $H(\alpha) = T(\alpha) \cup F'(\alpha) = U \cup F'(\alpha) = U$. Thus, $(H.T) = U_T$. That is, the left-absorbing element of λ_R in $S_T(U)$ is the soft set U_T.

$(F,T)\lambda_R U_M = (F, T \cap M)$.

Proof: let $U_M = (K,M)$. Thus, for all $\alpha \in M$, $K(\alpha) = U$. Let $(F,T)\lambda_R(K,M) = (H, T \cap M)$, where for all $\alpha \in T \cap M$, $H(\alpha) = F(\alpha) \cup T'(\alpha) = F(\alpha) \cup \emptyset = F(\alpha)$. Thus, $(H, T \cap M) = (F, T \cap M)$.

$U_M \lambda_R (F,T) = U_{T\cap M}$.

Proof: let $U_M=(K,M)$, where for all $\alpha\in M$, $K(\alpha)=U$. Let $(K,M)\,\lambda_R(F,T)=(H,M\cap T)$, where for all $\alpha\in M\cap T$, $H(\alpha)=T(\alpha)\cup F'(\alpha)=U\cup F'(\alpha)=U$. Thus, $(H, M\cap T)= U_{T\cap M}$.

$(F,T)\ \lambda_R\ U_E=(F,T)$.

Proof: let $U_E=(K,E)$, where for all $\alpha\in E$, $K(\alpha)=U$. Let $(F.T)\ \lambda_R(K.E)=(H.T\cap E)$, where for all $\alpha\in T\cap E=T$. $H(\alpha)=F(\alpha)\cup K'(\alpha)=F(\alpha)\cup\emptyset=F(\alpha)$. Thus $(H.T)=(F.T)$. That is, the right identity element of λ_R in $S_T(U)$ is the soft set U_E.

$U_E\lambda_R(F.T)=U_T$,

Proof: let $U_E=(KE)$, where for all $\alpha\in E$, $K(\alpha)=U$. Let $(K,E)\ \lambda_R(F.T)=(H.E\cap T)$, where for all $\alpha\in E\cap T=T$, $H(\alpha)=T(\alpha)\cup F'(\alpha)=U\cup F'(\alpha)=U$. Thus $(H,T)=U_T$.

$(F.T)\lambda_R(F.T)^r=(F.T)$,

Proof: let $(F.T)^r=(H.T)$, where for all $\alpha\in T$, $H(\alpha)=F'(\alpha)$. Let $(F.T)\lambda_R(H.T)=(L.T\cap T)$, where for all $\alpha\in T$, $L(\alpha)=F(\alpha)\cup H'(\alpha)=F(\alpha)\cup F(\alpha)=F(\alpha)$. Thus, $(L.T)=(F.T)$. That is, every relative complement of the soft set is its own right identity element for λ_R in $S_E(U)$.

$(F.T)^r\ \lambda_R(F.T)=(F.T)^r$,

Proof: let $(F.T)^r=(H.T)$, where for all $\alpha\in T$, $H(\alpha)=F'(\alpha)$. Let $(H.T)\lambda_R(F.T)=(L.T\cap T)$, where for all $\alpha\in T$, $T(\alpha)=H(\alpha)\cup F'(\alpha)=F'(\alpha)\cup F'(\alpha)=F'(\alpha)$. Thus $(L.T)=(F.T)^r$. That is, every relative complement of the soft set is its own left-absorbing element for λ_R in $S_E(U)$.

$[(F.T)\lambda_R(G.Z)]^r=(F.T)\gamma_R(G.Z)$,

Proof: let $(F.T)\lambda_R(G.Z)=(H.T\cap Z)$, where for all $\alpha\in T\cap Z$, $H(\alpha)=F(\alpha)\cup G'(\alpha)$. Let $(H.T\cap Z)^r=(K.T\cap Z)$, where for all $\alpha\in T\cap Z$, $K(\alpha)=F'(\alpha)\cap G(\alpha)$. Thus, $(K.T\cap Z)=(F.T)\gamma_R(G.Z)$. Here, if $T\cap Z=\emptyset$, then both side is the soft set $\emptyset_\emptyset$, and so the equality is again satisfied.

$(F.T)\lambda_R(G.T)=\emptyset_T \Leftrightarrow (F.T)=\emptyset_T$ and $(G.T)=U_T$,

Proof: let $(F.T)\lambda_R(G.T)=(K.T\cap T)$, where for all $\alpha\in T$, $K(\alpha)=F'(\alpha)\cup G(\alpha)$. Since $(K.T)=\emptyset_T$, for all $\alpha\in T$, $K(\alpha)=\emptyset$. Thus, for all $\alpha\in T$, $K(\alpha)=F(\alpha)\cup G'(\alpha)=\emptyset \Leftrightarrow$ for all $\alpha\in T$, $F(\alpha)=\emptyset$ and $G'(\alpha)=\emptyset \Leftrightarrow$ for all $\alpha\in T$, $F(\alpha)=\emptyset$ and $G(\alpha)=U \Leftrightarrow (F.T)=\emptyset_T$ and $(G.T)=U_T$.

$\emptyset_{T\cap Z} \mathrel{\tilde\subseteq} (F.T)\lambda_R(G.Z)$ and $(F.T)\lambda_R(G,Z) \mathrel{\tilde\subseteq} U_T$ and $(F,T)\lambda_R(G,Z) \mathrel{\tilde\subseteq} U_Z$,

$(F.T\cap Z) \mathrel{\tilde\subseteq} (F.T)\lambda_R(G.Z)$ and $(G.T\cap Z)^r \mathrel{\tilde\subseteq} (F.T)\lambda_R(G.Z)$,

Proof: let $(F.T)\lambda_R(G.Z)=(H.T\cap Z)$, where for all $\alpha\in T\cap Z$, $H(\alpha)=F\cup G'(\alpha)$. Since, for all $\alpha\in T\cap Z$, $F(\alpha)\subseteq F(\alpha)\cup G'(\alpha)=H(\alpha)$ and $G'(\alpha)\subseteq F(\alpha)\cup G'(\alpha)=H(\alpha)$. Thus, $(F.T\cap Z \mathrel{\tilde\subseteq} (F.T)\ \lambda_R(G,Z)$ and $(G,T\cap Z)^r \mathrel{\tilde\subseteq} (F.T)\ \lambda_R(G,Z)$.

$(F.T) \mathrel{\tilde\subseteq} (F.T)\lambda_R(G.T)$ and $(G.T)^r \mathrel{\tilde\subseteq} (F.T)\lambda_R(G.T)$,

Proof: let $(F.T)\ \lambda_R(G.T)=(H.T\cap T)$, where for all $\alpha\in T$, $H(\alpha)=F(\alpha)\cup G'(\alpha)$. Since for all $\alpha\in T$, $F(\alpha)\subseteq F(\alpha)\cup G'(\alpha)=H(\alpha)$ Thus $(F.T) \mathrel{\tilde\subseteq} (F.T)\lambda_R(G.T)$. Similarly, since $G'(\alpha)\subseteq F(\alpha)\cup G'(\alpha)=H(\alpha)$ Thus $(G.T)^r \mathrel{\tilde\subseteq} (F.T)\lambda_R(G.T)$.

If $(F.T) \mathrel{\tilde\subseteq} (G.K)$. then $(F.T)\ \lambda_R\ (H.Z) \mathrel{\tilde\subseteq} (G.K)\ \lambda_R\ (H.Z)$ and $(H.Z)\ \lambda_R\ (G.T) \mathrel{\tilde\subseteq} (H.Z)\ \lambda_R\ (F.T)$,

Proof: let $(F.T) \mathrel{\tilde\subseteq} (G.T)$. Then, for all $\alpha\in T$, $F(\alpha)\subseteq G(\alpha)$. Let $(F.T)\ \lambda_R\ (H.Z)=(W.T\cap Z)$, where for all $\alpha\in T\cap Z$, $W(\alpha)=F(\alpha)\cup H'(\alpha)$. Let $(G.K)\ \lambda_R\ (H.Z)=(L.T\cap Z)$, where for all $\alpha\in T\cap Z$, $L(\alpha)=F(\alpha)\cup H'(\alpha)$. Since for all $\alpha\in T\cap Z$, $(F.T)\ \lambda_R(H.Z) \mathrel{\tilde\subseteq} (G.K)\ \lambda_R\ (H.Z)$ and $H(\alpha)\cup G'(\alpha)\subseteq H(\alpha)\cup F'(\alpha)$. Also, since for all

$\alpha \in Z \cap T$, $H(\alpha) \cup G'(\alpha) \subseteq H(\alpha) \cup F'(\alpha)$, $(H.Z)\ \lambda_R\ (G.T) \cong\!\!\subseteq (H.Z)\ \lambda_R\ (F.T)$. Here, if $T \cap Z = \emptyset$, then both side is the soft set $\emptyset_\emptyset$, and so the property is again satisfied.

If $(F.T)\ \lambda_R (H.Z) \cong\!\!\subseteq (G.K)\ \lambda_R\ (H.Z)$, then $(F.T) \cong\!\!\subseteq (G.K)$ needs not be true. That is, the converse of *Theorem 1* is not true. Similarly, if $(H.Z)\ \lambda_R\ (G.T) \cong\!\!\subseteq (H.Z)\ \lambda_R\ (F.T)$, $(F.T) \cong\!\!\subseteq (G.K)$ needs not be true.

Proof: we give a counterexample to show that the onverse of *Theorem 1* is not true. Let $E = \{e_1.e_2.e_3.e_4.e_5\}$ be the parameter set, $T = \{e_1.e_3\}$, K=$\{e_1.e_3.e_5\}$, and $Z = \{e_1.e_3.e_5.e_6\}$ be the subsets of E, $U = \{h_1.h_2.h_3.h_4.h_5\}$ be the universal set, and (F,T), (G,K) and (H,Z) be the soft sets as follows:

$(F,T)=\{(e_1.\{h_2.h_5,\}),(e_3,\{h_1,h_2,h_5,\})\}$.

$(G,K)=\{(e_{1,}.\{h_2\}),(e_3,\{h_1,h_2\}).(e_5,U)\}$.

$(H,Z)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,U),(e_6,U)\}$.

Let $(F,T)\ \lambda_R(H,Z)=(L,T\cap Z)$, where for all $\alpha \in T \cap Z=\{e_1.e_3\}$, $L(\alpha)=F(\alpha)\cup H'(\alpha)$, $L(e_1)=F(e_1)\cup H'(e_1)=U$, $L(e_3)=F(e_3)\cup H'(e_3)=U$. Thus, $(F,T)\ \lambda_R\ (H,Z) = \{(e_1,U), (e_3,U)\}$.

Now let $(G,K)\ \lambda_R\ (H,Z)=(K,K\cap Z)$, where for all $\alpha \in K \cap Z = \{e_1, e_3, e_5\}$, $K(\alpha)=G(\alpha) \cup H'(\alpha)$, $K(e_1)=G(e_1)\cup H'(e_1)=U$, $K(e_3)=G(e_3)\cup H'(e_3)=U$, $W(e_5)=F(e_5)\cup H'(e_5)=U$. Thus, $(G,K)\ \lambda_R\ (H,Z)=\{(e_1,U), (e_3,U), (e_5,U)\}$.

It is observed that $(F,T)\ \lambda_R\ (H,Z) \cong\!\!\subseteq (G,K)\ \lambda_R (H,Z)$; however, (F,T) is not a soft subset of (G,K). Similarly, one can show that if $(H,Z)\ \lambda_R\ (G,T) \cong\!\!\subseteq (H,Z)\ \lambda_R\ (F,T)$, then $(F,T) \cong\!\!\subseteq (G,K)$ needs not to be true.

If $(F,T) \cong\!\!\subseteq (G.T)$ and $(K,V) \cong\!\!\subseteq (L.V)$, $(F,T)\ \lambda_R(L,V) \cong\!\!\subseteq (G,T)\ \lambda_R\ (K,V)$. Similarly, $(K,V)\ \lambda_R(G,T) \cong\!\!\subseteq (L,V)\ \lambda_R\ (F,T)$.

Proof: let $(F,T) \cong\!\!\subseteq (G.T)$ and $(K,V) \cong\!\!\subseteq (L.V)$, Thus, for all $\alpha\in T$ and for all $\alpha\in Z$, $F(\alpha)\subseteq G(\alpha)$ and $K(\alpha)\subseteq L(\alpha)$. Hence, for all $\alpha\in T$, $G'(\alpha)\subseteq F'(\alpha)$ and for all $\alpha\in Z$, $L'(\alpha)\subseteq K'(\alpha)$. Let $(F,T)\ \lambda_R\ (L,V)=(M,T\cap V)$. Thus, for all $\alpha\in T\cap V$, $M(\alpha)=F(\alpha)\cup L'(\alpha)$. Let $(G,T)\ \lambda_R\ (K,V)=(N,T\cap V)$. Thus, for all $\alpha\in T\cap V$, $N(\alpha)=G(\alpha)\cup K'(\alpha)$. Since, for all $\alpha\in T\cap V$, $F(\alpha)\subseteq G(\alpha)$ and $L'(\alpha)\subseteq K'(\alpha)$, $M(\alpha)=F(\alpha)\cup L'(\alpha)\subseteq G(\alpha) \cup K'(\alpha) = N(\alpha)$. Thus, $(F,T)\ \lambda_R(L,V) \cong\!\!\subseteq (G,T)\ \lambda_R\ (K,V)$. Under similar conditions, since for all $\alpha \in V \cap T$, $K(\alpha) \cup G'(\alpha) \subseteq L(\alpha) \cup F'(\alpha)$ and $(K.V)\lambda_R(G.T) \cong\!\!\subseteq (L.V)\ \lambda_R(F.T)$ can be illustrated similarly. Here, if $T \cap V = \emptyset$, then both side is the soft set $\emptyset_\emptyset$, and so the property is again satisfied.

Theorem 2. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted lambda operation distributes over other restricted soft set operations as follows:

I. LHS Distributions:

$(F,T)\ \lambda_R\ [(G,Z)\cap_R(H,M)] = [(F,T)\ \lambda_R(G,Z)] \cup_R [(F,T)\ \lambda_R(H,M)]$.

Proof: consider first the LHS. Let $(G,Z)\cap_R(H,M)=(R,Z\cap M)$, where for all $\alpha\in Z\cap M$, $R(\alpha)=G(\alpha)\cap H(\alpha)$. Let $(F,T)\ \lambda_R(R,Z\cap M)=(N,T\cap(Z\cap M))$, where for all $\alpha\in T\cap(Z\cap M)$, $N(\alpha)= F(\alpha)\cup R'(\alpha)$. Thus, for all $\alpha\in T\cap Z\cap M$,

$N(\alpha)= F(\alpha)\cup[(G'(\alpha)\cup H'(\alpha)]$.

Now consider the RHS, i.e. $[(F,T)\ \lambda_R(G,Z)] \cup_R [(F,T)\ \lambda_R(H,M)]$. Let $(F,T)\ \lambda_R\ (G,Z)=(V,T\cap Z)$, where for all $\alpha\in T\cap Z$, $V(\alpha)=F(\alpha)\cup G'(\alpha)$ and let $(F,T)\lambda_R\ (H,M)=(W,T\cap M)$, where for all $\alpha\in T\cap M$, $W(\alpha)=F(\alpha)\cup H'(\alpha)$. Let $(V,T\cap Z) \cup_R(W,T\cap M)=(S,(T\cap Z)\cap(T\cap M))$, where for all $\alpha\in T\cap Z\cap M$, $S(\alpha)=V(\alpha)\cup W(\alpha)$. Thus,

$S(\alpha)= [F(\alpha)\cup G'(\alpha)]\cup[F(\alpha)\cup H'(\alpha)]$.

Hence, $(N,T\cap Z\cap M)=(S,T\cap Z\cap M)$. Here, if $T\cap Z=\emptyset$ or $T\cap M=\emptyset$ or $Z\cap M=\emptyset$, then both sides is $\emptyset_\emptyset$. Thus, the equality is satisfied in all circumstances.

$(F,T)\ \lambda_R[(G,Z) \cup_R (H,M)] = [(F,T)\ \lambda_R(G,Z)] \cap_R [(F,T)\ \lambda_R\ (H,M)]$.

$(F,T)\ \lambda_R\ [(G,Z)\ \theta_R\ (H,M)] = [(F,T) \cup_R(G,Z)]\ \cup_R [(F,T) \cup_R\ (H,M)]$.

$(F,T)\ \lambda_R\ [(G,Z)\ *_R\ (H,M)] = [(F,T)\ \cup_R (G,Z)]\ \cap_R\ [(F,T)\ \cup_R\ (H,M)]$.

II. RHS Distributions:

$[(F,T)\ \cup_R\ (G,Z)]\ \lambda_R (H,M) = [(F,T)\ \lambda_R (H,M)]\ \cup_R\ [(G,Z)\ \lambda_R\ (H,M)]$.

Proof: consider first the LHS. Let $(F,T)\cup_R(G,Z)=(R,T\cap Z)$, where for all $\alpha\in T\cap Z$, $R(\alpha)=F(\alpha)\cup G(\alpha)$. Let $(R,T\cap Z)\ \lambda_R(H,M)=(N,(T\cap Z)\cap M))$, where for all $\alpha\in(T\cap Z)\cap M$, $N(\alpha)=R(\alpha)\cup H'(\alpha)$. Thus,

$N(\alpha)= [F(\alpha)\cup G(\alpha)]\cup H'(\alpha)$.

Now consider the RHS, i.e. $[(F,T)\ \lambda_R(H,M)]\ \cup_R\ [(G,Z)\ \lambda_R\ (H,M)]$. Let $(F,T)\lambda_R(H,M)=(S,T\cap M)$, where for all $\alpha\in T\cap M$, $T(\alpha)=F(\alpha)\cup H'(\alpha)$ and let $(G,Z)\ \lambda_R(H,M)= (K,Z\cap M)$, where for all $\alpha\in Z\cap M$, $K(\alpha)=G(\alpha)\cup H'(\alpha)$. Assume that $(S,T\cap Z)\ \cup_R(K,Z\cap M)=(L,(T\cap Z\cap M))$, where for all $\alpha\in(T\cap Z)\cap(Z\cap M)$, $L(\alpha)=S(\alpha)\cup K(\alpha)$. Thus,

$L(\alpha)=[\ F(\alpha)\cup H'(\alpha)]\ \cup\ [G(\alpha)\cup H'(\alpha)]$.

Hence, $(N,T\cap Z\cap M)=(L,T\cap Z\cap M)$. Here, if $T\cap Z=\emptyset$ or $T\cap M=\emptyset$ or $Z\cap M=\emptyset$, then both sides is $\emptyset_\emptyset$. Thus, the equality is satisfied in all circumstances.

$[(F,T)\ \cap_\varepsilon\ (G,Z)]\ \lambda_R\ (H,M)=[(F,T)\ \lambda_R(H,M)]\ \cap_R[(G,Z)\ \lambda_R(H,M)]$.

$[(F,T)\ \theta_R\ (G,Z)]\ \lambda_R\ (H,M)= [(F,T)\ *_R(H,M)]\ \cap_R\ [(G,Z)\ *_R(H,M)]$.

$[(F,T)\ *_R(G,Z)]\ \lambda_R(H,M)= [(F,T)\ *_R(H,M)]\ \cup_R\ [(G,Z)\ *_R\ (H,M)]$.

Theorem 3. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted lambda operation distributes over extended soft set operations as follows:

I. LHS Distributions:

$(F,T)\ \lambda_R[(G,Z)\ \cap_\varepsilon(H,M)] = [(F,T)\ \lambda_R(G,Z)]\ \cup_\varepsilon\ [(F,T)\ \lambda_R\ (H,M)]$.

Proof: consider first the LHS. Let $(G,Z)\ \cap_\varepsilon\ (H,M)=(R,Z\cup M)$, where for all $\alpha\in Z\cup M$,

$$R(\alpha) = \begin{cases} G(\alpha). & \alpha \in Z - M, \\ H(\alpha). & \alpha \in M - Z, \\ G(\alpha) \cap H(\alpha). & \alpha \in Z \cap M, \end{cases}$$

Let $(F,T)\lambda_R(R,Z\cup M)=(N,(T\cap(Z\cup M))$, where for all $\alpha\in T\cap(Z\cup M)$, $N(\alpha)=F'(\alpha)\cup R(\alpha)$. Thus,

$$N(\alpha)=\begin{cases} F(\alpha) \cup G'(\alpha). & \alpha \in T \cap (Z - M) = T \cap Z \cap M', \\ F(\alpha) \cup H'(\alpha). & \alpha \in T \cap (M - Z) = T \cap Z' \cap M, \\ F(\alpha) \cup [G'(\alpha) \cup H'(\alpha)]. & \alpha \in T \cap (Z \cap M) = T \cap Z \cap M, \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\ \lambda_R(G,Z)]\ \cup_\varepsilon\ [(F,T)\ \lambda_R\ (H,M)]$. Let $(F,T)\lambda_R(G,Z)=(K,T\cap Z)$, where for all $\alpha\in T\cap Z$, $K(\alpha)=F(\alpha)\cup G'(\alpha)$ and let $(F,T)\lambda_R(H,M)=(S,T\cap M)$, where for all $\alpha\in T\cap M$, $S(\alpha)= F(\alpha)\cup H'(\alpha)$. Let $(K,T\cap Z)\ \cup_\varepsilon(S,T\cap M)=(L,(T\cap Z)\cup(T\cap M))$, where for all $\alpha\in(T\cap Z)\cup(T\cap M)$,

$$L(\alpha) = \begin{cases} K(\alpha). & \alpha \in (T \cap Z) - (T \cap M) = T \cap (Z - M), \\ S(\alpha). & \alpha \in (T \cap M) - (T \cap Z) = T \cap (M - Z), \\ K(\alpha) \cup S(\alpha). & \alpha \in (T \cap Z) \cap (T \cap M) = T \cap (Z \cap M), \end{cases}$$

Thus,

$$L(\alpha) = \begin{cases} F(\alpha) \cup G'(\alpha). & \alpha \in T \cap Z \cap M', \\ F(\alpha) \cup H'(\alpha). & \alpha \in T \cap Z' \cap M, \\ [F(\alpha) \cup G'(\alpha)] \cup [F(\alpha) \cup H'(\alpha)]. & \alpha \in T \cap Z \cap M, \end{cases}$$

Hence, (N,T∩(Z∪M))=(L,(T∩Z)∪(T∩M)). Here, if T∩Z=∅, then N(α)=L(α)=F(α)∪H'(α), and if T∩M=∅, then N(α)=L(α)=F(α)∪G'(α). Thus, there is no extra condition as T∩Z≠ ∅ and/or T∩M≠ ∅ for satisfying *Theorem 3*.

$$(F,T)\ \lambda_R[(G,Z)\ \cup_\varepsilon\ (H,M)] = [(F,T)\ \lambda_R(G,Z)]\ \cap_\varepsilon[(F,T)\ \lambda_R\ (H,M)].$$

II. RHS Distributions:

$$[(F,T)\cup_\varepsilon(G,Z)]\ \lambda_R(H,M)= [(F,T)\ \lambda_R(H,M)]\ \cup_\varepsilon\ [(G,Z)\ \lambda_R\ (H,M)].$$

Proof: consider first the LHS. Let $(F,T)\ \cup_\varepsilon(G,Z)=(R,T\cup Z)$, where for all α∈T∪Z,

$$R(\alpha) = \begin{cases} F(\alpha). & \alpha \in T - Z, \\ G(\alpha). & \alpha \in Z - T, \\ F(\alpha) \cup G(\alpha). & \alpha \in T \cap Z, \end{cases}$$

Assume that $(R,T\cup Z)\ \lambda_R(H,M)=(N,(T\cup Z)\cap M))$, where for all α∈(T∪Z)∩M, N(α)=R(α)∪H' (α). Thus,

$$N(\alpha) = \begin{cases} F(\alpha) \cup H'(\alpha). & \alpha \in (T - Z) \cap M = T \cap Z' \cap M, \\ G(\alpha) \cup H'(\alpha). & \alpha \in (Z - T) \cap M = T' \cap Z \cap M, \\ [F(\alpha) \cup G(\alpha)] \cup H'(\alpha). & \alpha \in (T \cap Z) \cap M = T \cap Z \cap M, \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\ \lambda_R(H,M)]\ \cup_\varepsilon\ [(G,Z)\ \lambda_R\ (H,M)]$. Let $(F,T)\ \lambda_R\ (H,M)=(K,T\cap M)$, where for all α∈T∩M, K(α)=F(α)∪H'(α) and let $(G,Z)\ \lambda_R\ (H,M)=(S,Z\cap M)$, where for all α∈Z∩M, S(α)=G(α)∪H'(α). Let $(K,T\cap M)\ \cup_\varepsilon(S,Z\cap M)=(L,(\ (T\cap M)\cup(Z\cap M))$. Hence,

$$L(\alpha) = \begin{cases} K(\alpha). & \alpha \in (T \cap M) - (Z \cap M) = (T - Z) \cap M, \\ S(\alpha). & \alpha \in (Z \cap M) - (T \cap M) = (Z - T) \cap M, \\ K(\alpha) \cup S(\alpha). & \alpha \in (T \cap M) \cap (Z \cap M) = (T \cap Z) \cap M, \end{cases}$$

Thus,

$$L(\alpha) = \begin{cases} F(\alpha) \cup H'(\alpha). & \alpha \in T \cap Z' \cap M, \\ G(\alpha) \cup H'(\alpha). & \alpha \in T' \cap Z \cap M, \\ [F(\alpha) \cup H'(\alpha)] \cup [G(\alpha) \cup H'(\alpha)]. & \alpha \in T \cap Z \cap M, \end{cases}$$

Therefore, (N,(T∪Z)∩M)) = (L,(T∩M)∪(Z∩M)).

Here, if T∩Z=∅ and α∈T∩Z'∩M, then N(α)=L(α)=F'(α)∪H(α) and if T∩Z=∅ and α∈T'∩Z∩M, the N(α)=L(α)=G(α)∪H'(α). Furthermore, if Z∩M=∅, then N(α)=L(α)=F(α)∪H'(α). Thus, there is no extra condition as T∩Z≠ ∅ and/or Z∩M≠ ∅ for satisfying *Theorem 3.*

$$[(F,T)\ \cap_\varepsilon\ (G,Z)]\ \lambda_R(H,M)= [(F,T)\ \lambda_R(H,M)]\ \cap_\varepsilon\ [(G,Z)\ \lambda_R\ (H,M)].$$

Theorem 4. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted lambda operation distributes over complementary extended soft set operations as follows:

I. LHS Distributions:

$$(F,T)\ \lambda_R[(G,Z)\ \overset{*}{*}_\varepsilon\ (H,M)] = [(F,T)\cup_R(G,Z)]\ \cap_\varepsilon[(F,T)\ \cup_R\ (H,M)].$$

Proof: consider first the LHS. Let $(G,Z)\ \overset{*}{*}_\varepsilon(H,M)=(R,Z\cup M)$, where for all α∈Z∪M,

$$R(\alpha) = \begin{cases} G'(\alpha). & \alpha \in Z - M, \\ H'(\alpha). & \alpha \in M - Z, \\ G'(\alpha) \cup H'(\alpha). & \alpha \in Z \cap M, \end{cases}$$

Let $(F,T)\lambda_R(R,Z\cup M)=(N,(T\cap(Z\cup M))$, where for all α∈T∩(Z∪M), N(α)=F(α)∪R' (α). Thus,

$$N(\alpha)=\begin{cases}F(\alpha)\cup G(\alpha). & \alpha\in Z-M,\\ F(\alpha)\cup H(\alpha). & \alpha\in M-Z,\\ F(\alpha)\cup[G(\alpha)\cap H(\alpha)]. & \alpha\in Z\cap M,\end{cases}$$

Now consider the RHS, i.e. $(F,T)\cup_R(G,Z)=(K,T\cap Z)$ where for all $\alpha\in T\cap Z$, $K(\alpha)=F(\alpha)\cup G(\alpha)$. Let $(F,T)\cup_R(H,M)=(S,T\cap M)$, where for all $\alpha\in T\cap M$, $S(\alpha)=F(\alpha)\cup H(\alpha)$. Assume that $(K,T\cap Z)\cap_\varepsilon(S,T\cap M)=(L,(T\cap Z)\cup(T\cap M))$, where for all $\alpha\in(T\cap Z)\cup(T\cap M)$,

$$L(\alpha)=\begin{cases}K(\alpha). & \alpha\in(T\cap Z)-(T\cap M)=T\cap(Z-M),\\ S(\alpha). & \alpha\in(T\cap M)-(T\cap Z)=T\cap(M-Z),\\ K(\alpha)\cap S(\alpha). & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap(Z\cap M),\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases}F(\alpha)\cup G(\alpha). & \alpha\in T\cap Z\cap M',\\ F(\alpha)\cup H(\alpha). & \alpha\in T\cap Z'\cap M,\\ [F(\alpha)\cup G(\alpha)]\cap[F(\alpha)\cup H(\alpha)]. & \alpha\in T\cap Z\cap M,\end{cases}$$

Therefore, $(N,(T\cap(Z\cup M))=(L,(T\cap Z)\cup(T\cap M))$. Here, if $T\cap Z=\emptyset$, then $N(\alpha)=L(\alpha)=F(\alpha)\cup H(\alpha)$, and if $T\cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F(\alpha)\cup G(\alpha)$. Thus, there is no extra condition as $T\cap Z\neq\emptyset$ and/or $T\cap M\neq\emptyset$ for satisfying *Theorem 3*.

$$(F,T)\,\lambda_R\,[(G,Z)\overset{*}{\theta_\varepsilon}(H,M)]=[(F,T)\cup_R(G,Z)]\cup_R[(F,T)\cup_R(H,M)].$$

II. RHS Distributions:

$$[(F,T)\overset{*}{\theta_\varepsilon}(G,Z)]\,\lambda_R(H,M)=[(F,T)*_R(H,M)]\cap_\varepsilon[(G,Z)*_R(H,M)].$$

Proof: consider first the LHS. Let $(F,T)\overset{*}{\theta_\varepsilon}(G,Z)=(R,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$R(\alpha)=\begin{cases}F'(\alpha). & \alpha\in T-Z,\\ G'(\alpha). & \alpha\in Z-T,\\ F'(\alpha)\cap G'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Let $(R,T\cup Z)\,\lambda_R(H,M)=(N,(T\cup Z)\cap M)$, where for all $\alpha\in(T\cup Z)\cap M$, $N(\alpha)=R'(\alpha)\cup H(\alpha)$. Thus,

$$N(\alpha)=\begin{cases}F'(\alpha)\cup H'(\alpha). & \alpha\in(T-Z)\cap M=T\cap Z'\cap M,\\ G'(\alpha)\cup H'(\alpha). & \alpha\in(Z-T)\cap M=T'\cap Z\cap M,\\ [F'(\alpha)\cap G'(\alpha)]\cup H'(\alpha). & \alpha\in(T\cap Z)\cap M=T\cap Z\cap M,\end{cases}$$

Now consider the RHS, i.e. $[(F,T)*_R(H,M)]\cap_\varepsilon[(G,Z)*_R(H,M)]$. Let $(F,T)*_R(H,M)=(K,T\cap M)$, where for all $\alpha\in T\cap M$, $K(\alpha)=F'(\alpha)\cup H'(\alpha)$ and let $(G,Z)*_R(H,M)=(S,Z\cap M)$, where for all $\alpha\in Z\cap M$, $S(\alpha)=G'(\alpha)\cup H'(\alpha)$. Assume that $(K,T\cap M)\cap_\varepsilon(S,Z\cap M)=(L,(T\cap M)\cup(Z\cap M))$, where for all $\alpha\in(T\cap M)\cup(Z\cap M)$,

$$L(\alpha)=\begin{cases}K(\alpha). & \alpha\in(T\cap M)-(Z\cap M)=(T-Z)\cap M,\\ S(\alpha). & \alpha\in(Z\cap M)-(T\cap M)=(Z-T)\cap M,\\ K(\alpha)\cap S(\alpha). & \alpha\in(T\cap M)\cap(Z\cap M)=(T\cap Z)\cap M,\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases}F'(\alpha)\cup H'(\alpha). & \alpha\in T\cap Z'\cap M,\\ G'(\alpha)\cup H'(\alpha). & \alpha\in T'\cap Z\cap M,\\ [F'(\alpha)\cup H'(\alpha)]\cap[G'(\alpha)\cup H'(\alpha)]. & \alpha\in T\cap Z\cap M,\end{cases}$$

Therefore, $(N,(T\cup Z)\cap M)=(L,(T\cap M)\cup(Z\cap M))$. Here, if $T\cap Z=\emptyset$ and $\alpha\in T\cap Z'\cap M$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cup H'(\alpha)$ and if $T\cap Z=\emptyset$ and $\alpha\in T'\cap Z\cap M$, the $N(\alpha)=L(\alpha)=G'(\alpha)\cup H'(\alpha)$. Furthermore, if

$Z \cap M = \emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha) \cup H'(\alpha)$. Thus, there is no extra condition as $T \cap Z \neq \emptyset$ and/or $Z \cap M \neq \emptyset$ for satisfying *Theorem 4*.

$$[(F,T) \overset{*}{*}_{\varepsilon} (G,Z)]\ \lambda_R (H,M)= [(F,T) *_R (H,M)] \cup_{\varepsilon} [(G,Z) *_R (H,M)].$$

Theorem 5. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted lambda operation distributes over soft binary piecewise operations as follows:

I. LHS distributions:

$$(F,T)\ \lambda_R [(G,Z) \underset{\cap}{\sim} (H,M)]=[(F,T)\ \lambda_R (G,Z)] \underset{\cup}{\sim} [(F,T)\ \lambda_R\ (H,M)].$$

Proof: consider first the LHS. Let $(G,Z) \underset{\cap}{\sim} (H,M)=(R,Z)$, where for all $\alpha \in Z$,

$$R(\alpha) = \begin{cases} G(\alpha). & \alpha \in Z - M, \\ G(\alpha) \cap H(\alpha). & \alpha \in Z \cap M, \end{cases}$$

Let $(F,T)\ \lambda_R\ (R,Z) =(N, T \cap Z)$, where for all $\alpha \in T \cap Z$, $N(\alpha)=F(\alpha) \cup R'(\alpha)$. Thus,

$$N(\alpha) = \begin{cases} F(\alpha) \cup G'(\alpha). & \alpha \in T \cap (Z - M) = T \cap Z \cap M', \\ F(\alpha) \cup [G'(\alpha) \cup H'(\alpha)]. & \alpha \in T \cap (Z \cap M) = T \cap Z \cap M, \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\ \lambda_R (G,Z)] \underset{\cup}{\sim} [(F,T)\ \lambda_R\ (H,M)]$. Let $(F,T)\lambda_R(G,Z)=(K,T \cap Z)$, where for all $\alpha \in T \cap Z$, $K(\alpha)=F(\alpha) \cup G'(\alpha)$. Let $(F,T)\ \lambda_R\ (H,M)=(S,T \cap M)$, where for all $\alpha \in T \cap M$, $S(\alpha)=F(\alpha) \cup H'(\alpha)$ and assume that $(K,T \cap Z) \underset{\cup}{\sim} (S,T \cap M)=(L,T \cap Z)$, where for all $\alpha \in T \cap Z$,

$$L(\alpha) = \begin{cases} K(\alpha). & \alpha \in (T \cap Z) - (T \cap M) = T \cap (Z - M), \\ K(\alpha) \cup S(\alpha). & \alpha \in (T \cap Z) \cap (T \cap M) = T \cap (Z \cap M), \end{cases}$$

Thus,

$$L(\alpha) = \begin{cases} F(\alpha) \cup G'(\alpha). & \alpha \in T \cap Z \cap M', \\ [F(\alpha) \cup G'(\alpha)] \cup [F(\alpha) \cup H'(\alpha)]. & \alpha \in T \cap Z \cap M, \end{cases}$$

Hence, $(N. T \cap Z) = (L. T \cap Z)$. Here, if $T \cap Z = \emptyset$, then $(N,T \cap Z)=(L,T \cap Z)=\emptyset_{\emptyset}$, and if $T \cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F(\alpha) \cup G'(\alpha)$. Thus, there is no extra condition as $T \cap Z \neq \emptyset$ and/or $T \cap M \neq \emptyset$ for satisfying *Theorem 5*.

$$(F,T)\ \lambda_R [(G,Z) \underset{\cup}{\sim} (H,M)]=[(F,T)\lambda_R (G,Z)] \underset{\cap}{\sim} [(F,T)\ \lambda_R\ (H,M)].$$

II. RHS distributions:

$$[(F,T) \underset{\cup}{\sim} (G,Z)]\ \lambda_R (H,M)= [(F,T)\ \lambda_R (H,M)] \underset{\cup}{\sim} [(G,Z)\ \lambda_R\ (H,M)].$$

Proof: consider first the LHS. Let $(F,T) \underset{\cup}{\sim} (G,Z)=(R,T)$, where for all $\alpha \in T$,

$$R(\alpha) = \begin{cases} F(\alpha). & \alpha \in T - Z, \\ F(\alpha) \cup G(\alpha). & \alpha \in T \cap Z, \end{cases}$$

Let $(R,T)\ \lambda_R\ (H,M) =(N,T \cap M)$, where for all $\alpha \in T \cap M$, $N(\alpha)=R'(\alpha) \cup H(\alpha)$. Thus,

$$N(\alpha) = \begin{cases} F(\alpha) \cup H'(\alpha). & \alpha \in (T - Z) \cap M = T \cap Z' \cap M, \\ [F(\alpha) \cup G(\alpha)] \cup H'(\alpha). & \alpha \in (T \cap Z) \cap M = T \cap Z \cap M, \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\ \lambda_R (H,M)] \underset{\cup}{\sim} [(G,Z)\ \lambda_R\ (H,M)]$. Let $(F,T)\lambda_R(H,M)=(K,T \cap M)$, where for all $\alpha \in T \cap M$, $K(\alpha)=F(\alpha) \cup H'(\alpha)$. Assume that $(G,Z)\ \lambda_R\ (H,M)=(S,Z \cap M)$, where for all $\alpha \in Z \cap M$, $S(\alpha)=G(\alpha) \cup H'(\alpha)$ and let $(K,T \cap M) \underset{\cup}{\sim} (S,Z \cap M)=(L,T \cap M)$, where for all $\alpha \in T \cap M$,

$$L(\alpha) = \begin{cases} K(\alpha). & \alpha \in (T \cap M) - (Z \cap M) = (T - Z) \cap M, \\ K(\alpha) \cup S(\alpha). & \alpha \in (T \cap M) \cap (Z \cap M) = (T \cap Z) \cap M, \end{cases}$$

Hence,

$$L(\alpha)=\begin{cases}F(\alpha)\cup H'(\alpha) & \alpha\in T\cap Z'\cap M,\\ [F(\alpha)\cup H'(\alpha)]\cup[G(\alpha)\cup H'(\alpha)] & \alpha\in T\cap Z\cap M,\end{cases}$$

Thus, (N,T∩M)=(L,T∩M). Here, if T∩M=∅, then (N,T∩M)=(L,T∩M)=$\emptyset_\emptyset$, and if Z∩M=∅, then N(α)=L(α)=F(α)∪H'(α). Thus, there is no extra condition as T∩M≠ ∅ and/or Z∩M≠ ∅ for satisfying *Theorem 5*.

$$[(F,T)\ \tilde{\cap}\ (G,Z)]\ \lambda_R(H,M)=[(F,T)\ \lambda_R\ (H,M)]\ \tilde{\cap}\ [(G,Z)\ \lambda_R\ (H,M)].$$

3.2. Extended Lambda Operation and its Properties

Definition 12. let (F. T) and (G. Z) be soft sets over U. The extended lambda operation of (F. T) and (G. Z) is the soft set (H,C), denoted by (F,T) λ_ε(G,Z)=(H,C), where C=T∪Z and for all α∈C,

$$H(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

From the definition, it is obvious that if T=∅, then (F,T)λ_ε(G,Z)=(G,Z), if Z=∅, then (F,T)λ_ε(G,Z)=(F,T), if T=Z=∅, then (F,T)λ_ε(G,Z)= $\emptyset_\emptyset$.

Example 2. let $E=\{e_1.e_2.e_3.e_4\}$be the parameter set, $T=\{e_1.e_3\}$ and $Z=\{e_2.e_3.e_4\}$ be subsets of E, $U=\{h_1.h_2.h_3.h_4.h_5\}$ be the universal set, (F. T) and (G. Z) be the soft sets over U as $(F.T)=\{(e_{1,}.\{h_2.h_5),(e_3.\{h_1.h_2.h_5\})\}$, $(G.Z)=\{(e_2.\{h_1.h_4.h_5\}),\{(e_3.\{h_2.h_3.h_4\}),(e_4.h_3,h_5\})\}$. Here, let (F,T) λ_ε(G,Z)=(H,T∪Z), where for all α∈T∪Z,

$$H(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Since T∪Z=$\{e_1,e_2,e_3,e_4\}$ and T-Z=$\{e_1\}$, Z-T=$\{e_2.e_4\}$, T∩Z=$\{e_3\}$, thus, $H(e_1)=F(e_1)=\{h_{2,}h_5\}$, $H(e_2)=G(e_2)=\{h_1,h_4,h_5\}$, $H(e_4)=G(e_4)=\{h_3,h_5\}$, $H(e_3)=F(e_3)\cup G',(e_3)=\{h_1,h_2,h_5\}\cup\{h_1.h_5\}=\{h_1.h_2.h_5\}$.
Thus,
$(F,T)\lambda_\varepsilon(G,Z)=\{(e_1,\{h_{2,}h_5\}),(e_2,\{h_1,h_4,h_5\}),(e_3.\{h_1.h_2.h_5\}),(e_4,\{h_3,h_5\})\}$.

Remark 1. In the set S_T(U), where T is a fixed subset of E, restricted and extended lambda operations coincide with each other. That is, (F,T) λ_ε(G,T)=(F,T) λ_R(G,T).

Theorem 6 (Algebraic Properties of the Operation). Let (F,T), (G,T), (H,T) (G,Z), (H,M), (K,V) and (L,V) be soft sets over U. Then,

I. The set S_E(U) and S_T(U) are closed under λ_ε.

Proof: it is clear that λ_ε is a binary operation in S_E(U). That is,
λ_ε: $S_E(U)$x $S_E(U)\rightarrow S_E(U)$.
((F,T), (G,Z)) → (F,T)λ_ε(G,Z)=(H,T∪Z).

Namely, when (F. T) and (G. Z) are soft set over U, then so (F,T) λ_ε (G,Z). Similarly, S_T(U) is closed under λ_ε. That is,

λ_ε: $S_T(U)$x $S_T(U)\rightarrow S_T(U)$.
((F,T), (G,T)) → (F, T)λ_ε(G, T)=(K,T∪ T)=(K,T).

Namely, λ_ε is a binary operation in S_T(U).

If T∩Z∩M=∅, then [(F,T) λ_ε (G,Z)] λ_ε (H,M) = (F,T) λ_ε [(G,Z) λ_ε (H,M)].

Proof: first, consider the LHS. Let $(F,T)\lambda_\varepsilon(G,Z)=(S,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$S(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Let $(S,T\cup Z)\ \lambda_\varepsilon(H,M)=(N,(T\cup Z)\cup M))$, where for all $\alpha\in(T\cup Z)\cup M$,

$$N(\alpha)=\begin{cases}S(\alpha). & \alpha\in(T\cup Z)-M,\\ H(\alpha). & \alpha\in M-(T\cup M),\\ S(\alpha)\cup H'(\alpha). & \alpha\in(T\cup Z)\cap M.\end{cases}$$

Thus,

$$M(\alpha)=\begin{cases}F(\alpha). & \alpha\in(T-Z)-M=T\cap Z'\cap M',\\ G(\alpha). & \alpha\in(Z-T)-M=T'\cap Z\cap M',\\ F(\alpha)\cup G'(\alpha). & \alpha\in(T\cap Z)-M=T\cap Z\cap M',\\ H(\alpha). & \alpha\in M-(T\cup Z)=T'\cap Z'\cap M,\\ F(\alpha)\cup H'(\alpha). & \alpha\in(T-Z)\cap M=T\cap Z'\cap M,\\ G(\alpha)\cup H'(\alpha). & \alpha\in(Z-T)\cap M=T'\cap Z\cap M,\\ [F(\alpha)\cup G'(\alpha)]\cup H'(\alpha). & \alpha\in(T\cap Z)\cap M=T\cap Z\cap M,\end{cases}$$

Now consider the RHS. Let $(G,Z)\ \lambda_\varepsilon(H,M)=(R,Z\cup M)$, where for all $\alpha\in Z\cup M$,

$$R(\alpha)=\begin{cases}G(\alpha). & \alpha\in Z-M,\\ H(\alpha). & \alpha\in M-Z,\\ G(\alpha)\cup H'(\alpha). & \alpha\in Z\cap M,\end{cases}$$

Let $(F,T)\ \lambda_\varepsilon(R,Z\cup M)=(L,(T\cup(Z\cup M))$, where for all $\alpha\in T\cup Z\cup M$,

$$L(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-(Z\cup M),\\ R(\alpha). & \alpha\in(Z\cup M)-T,\\ F(\alpha)\cup R'(\alpha). & \alpha\in T\cap(Z\cup M),\end{cases}$$

Hence,

$$N(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-(Z\cup M)=T\cap Z'\cap M',\\ G(\alpha). & \alpha\in(Z-M)-T=T'\cap Z\cap M',\\ H(\alpha). & \alpha\in(M-Z)-T=T'\cap Z'\cap M,\\ G(\alpha)\cup H'(\alpha). & \alpha\in(Z\cap M)-T=T'\cap Z\cap M,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap(Z-M)=T\cap Z\cap M',\\ F(\alpha)\cup H'(\alpha). & \alpha\in T\cap(M-Z)=T\cap Z'\cap M,\\ F(\alpha)\cup[G'(\alpha)\cap H(\alpha)]. & \alpha\in T\cap(Z\cap M)=T\cap Z\cap M,\end{cases}$$

It is observed that $(N,(T\cup Z)\cup M)=(L,T\cup(Z\cup M))$, where $T\cap Z\cap M=\emptyset$. That is, in $S_E(U)$, λ_ε is associative under certain conditions.

$[(F,T)\ \lambda_\varepsilon\ (G,T)]\ \lambda_\varepsilon\ (H,T)\neq(F,T)\ \lambda_\varepsilon\ [(G,T)\ \lambda\ (H,T)]$.

Proof: the proof follows from *Remark 1* and *Theorem 1*. That is, in $S_T(U)$, where T is a fixed subset of E, λ_ε is not associative.

$(F,T)\lambda_\varepsilon(G,Z)\neq(G,Z)\ \lambda_\varepsilon(F,T)$.

Proof: let $(F,T)\ \lambda_\varepsilon(G,Z)=(H,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$H(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Let (G,Z) λ_ε(F,T)=(S,Z∪T), where for all α∈Z∪T,

$$S(\alpha)=\begin{cases}G(\alpha). & \alpha\in Z-T,\\ F(\alpha). & \alpha\in T-Z,\\ G(\alpha)\cup F'(\alpha). & \alpha\in Z\cap T,\end{cases}$$

Thus, $(F.T)\ \lambda_\varepsilon\ (G.Z)\neq(G.Z)\ \lambda_\varepsilon(F.T)$. If $Z\cap T=\emptyset$, then $(F.T)\lambda_\varepsilon\ (G.Z)=(G.Z)\ \lambda_\varepsilon\ (F.T)$. Moreover, it is obvious that $(F.T)\ \lambda_\varepsilon\ (G.T)\neq(G.T)\lambda_\varepsilon(F.T)$. That is, in $S_E(U)$ and $S_T(U)$, λ_ε is not commutative.

(F,T) λ_ε(F,T)= U_T.

Proof: the proof follows from *Remark 1* and *Theorem 1*. That is, in $S_E(U)$, λ_ε is not idempotent.

(F,T) λ_ε $\emptyset_T$=U_T.

Proof: the proof follows from *Remark 1*and *Theorem 1*.

$\emptyset_T\lambda_\varepsilon$(F,T)=$(F,T)^r$.

Proof: the proof follows from *Remark 1* and *Theorem 1*.

(F,T)λ_ε $\emptyset_\emptyset$=(F,T).

Proof: let $\emptyset_\emptyset$=(S,∅) and (F,T) λ_ε(S, ∅)=(H,T∪∅), where for all α∈T∪∅=T,

$$H(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-\emptyset=T,\\ S(\alpha). & \alpha\in\emptyset-T=\emptyset,\\ F(\alpha)\cup S'(\alpha). & \alpha\in T\cap\emptyset=\emptyset,\end{cases}$$

Thus, for all α∈T, H(α)=F(α), (H,T)=(F,T).

$\emptyset_\emptyset$ λ_ε (F,T)=(F,T).

Proof: Let $\emptyset_\emptyset$ =(S,∅) and (S,∅) λ_ε(F,T)=(H, ∅∪T), where for all α∈∅∪T=T,

$$H(\alpha)=\begin{cases}S(\alpha). & \alpha\in\emptyset-T=\emptyset,\\ F(\alpha). & \alpha\in T-\emptyset=T,\\ S(\alpha)\cup F'(\alpha). & \alpha\in\emptyset\cap T=\emptyset,\end{cases}$$

Thus, for all α∈T, H(α)=F(α), (H,T)=(F,T).

By *Theorem 6*, we can conclude that in $S_E(U)$, the identity element of λ_ε is the soft set $\emptyset_\emptyset$. In classical set theory, it is well-known that A∪B=∅⟺A=∅ and B=∅. Thus, it is evident that in $S_E(U)$, we can not find $(G.K)\in S_E(U)$ such that $(F.T)\ \lambda_\varepsilon(G.K)=(G,K)\ \lambda_\varepsilon(F.T)=\emptyset_\emptyset$, as this situation requires that T ∪K=∅ and thus, T=∅ and K=∅. Since in $S_E(U)$, the only soft set with an empty parameter set is $\emptyset_\emptyset$, it follows that only the identity element $\emptyset_\emptyset$ has an inverse and its inverse is its own, as usual. Thus, in $S_E(U)$, any other element except $\emptyset_\emptyset$ does not have an inverse for the operation λ_ε.

Corollary 1. Let (F,T), (G,Z), and (H,M) be the elements of $S_E(U)$. By *Theorem 6*, $(S_E(U),\lambda_\varepsilon)$ is a noncommutative monoid whose identity is $\emptyset_\emptyset$ where T∩Z∩M=∅. Since $(S_A(U),\lambda_\varepsilon)$ does not have associative property, where A is a fixed subset of E; this algebraic structure can not be a semigroup.

(F,T) λ_ε U_T=(F,T).

Proof: the proof follows from *Remark 1* and *Theorem* 1. That is, U_T is the right identity element for λ_ε in $S_T(U)$.

$U_T\lambda_\varepsilon$ (F,T)= U_T,

Proof: the proof follows from *Remark* 1 and *Theorem* 1. That is, U_T is the left absorbing element for λ_ε in $S_T(U)$.

$U_E \lambda_\varepsilon (F,T) = U_E$.

Proof: let U_E=(T,E), where for all α∈E, T(α)=U. Assume that (F,T) λ_ε(T, E)=(H,T∪E), where for all α∈T∪E=E,

$$K(\alpha) = \begin{cases} T(\alpha). & \alpha \in E - T, \\ F(\alpha). & \alpha \in T - E, \\ T(\alpha) \cup F'(\alpha). & \alpha \in E \cap T\,, \end{cases}$$

Thus,

$$K(\alpha) = \begin{cases} U. & \alpha \in E - T = T', \\ F(\alpha). & \alpha \in T - E = \emptyset, \\ U. & \alpha \in T \cap E = T, \end{cases}$$

Hence, for all α ∈ E, H(α) = U, and so (H. E) = U_E. That is, U_E is the left absorbing element for λ_ε in $S_E(U)$. Here note that $U_E\lambda_\varepsilon$(F,T) ≠ U_E, that is U_E is not the right absorbing element for λ_ε in $S_E(U)$. In deed, let U_E=(T,E) and (T. E) λ_ε(F,T)= (K,T∪E), where for all α∈T∪E=E,

$$H(\alpha) = \begin{cases} F(\alpha). & \alpha \in T - E, \\ T(\alpha). & \alpha \in E - T, \\ F(\alpha) \cup T'(\alpha). & \alpha \in T \cap E, \end{cases}$$

Thus,

$$H(\alpha) = \begin{cases} F(\alpha). & \alpha \in T - E = \emptyset, \\ U. & \alpha \in E - T = T', \\ F(\alpha). & \alpha \in T \cap E = T, \end{cases}$$

Hence, (K,E) ≠ U_E.

$(F,T)\lambda_\varepsilon (F,T)^r=(F,T)$.

Proof: the proof follows from *Remark 1* and *Theorem 1*. That is, every relative complement of the soft set is its own right identity element for the operation λ_ε in $S_E(U)$.

$(F,T)^r \lambda_\varepsilon(F,T)=(F,T)^r$.

Proof: the proof follows from *Remark 1* and *Theorem* 1. That is, every relative complement of the soft set is its own left absorbing element for the operation λ_ε in $S_E(U)$.

$[(F,T)\, \lambda_\varepsilon(G,Z)]^r=(F,T) \overset{*}{\underset{\gamma}{\sim}} (G,Z)$.

Proof: let (F,T) λ_ε(G,Z) =(H,T∪Z), where for all α∈T∪Z,

$$H(\alpha) = \begin{cases} F(\alpha). & \alpha \in T - Z, \\ G(\alpha). & \alpha \in Z - T, \\ F(\alpha) \cup G'(\alpha). & \alpha \in T \cap Z, \end{cases}$$

Let (H. T ∪ Z)r = (K. T ∪ Z), for all α ∈ T ∪ Z,

$$K(\alpha) = \begin{cases} F'(\alpha). & \alpha \in T - Z, \\ G'(\alpha). & \alpha \in Z - T, \\ F'(\alpha) \cap G(\alpha). & \alpha \in T \cap Z, \end{cases}$$

Thus, $(K, T \cup Z) = (F,T) \underset{\gamma}{\overset{*}{\sim}} (G,Z)$,

$(F,T)\ \lambda_\varepsilon(G,T) = \emptyset_T \Leftrightarrow (F,T) = \emptyset_T$ and $(G,T) = U$,

Proof: the proof follows from *Remark 1* and *Theorem* 1.

$\emptyset_T \mathrel{\tilde{\subseteq}} (F.T)\lambda_\varepsilon(G.Z)$, $\emptyset_Z \mathrel{\tilde{\subseteq}} (F.T)\ \lambda_\varepsilon\ (G.Z)$, $\emptyset_Z \mathrel{\tilde{\subseteq}} (G.Z)\ \lambda_\varepsilon\ (F.T)$, $\emptyset_T \mathrel{\tilde{\subseteq}}$ (G,Z) λ_ε (F,T). Moreover, (F,T) λ_ε (G,Z) $\mathrel{\tilde{\subseteq}} U_{T\cup Z}$ and (G,Z) λ_ε(F,T) $\mathrel{\tilde{\subseteq}}\ U_{Z\cup T}$.

(F,T) $\mathrel{\tilde{\subseteq}}$(F,T) λ_ε (G,T) and (G,T)^r $\mathrel{\tilde{\subseteq}}$ $(F,T)\lambda_\varepsilon$ (G,T),

Proof: the proof follows from *Remark 1* and *Theorem 1*.

If (F,T) $\mathrel{\tilde{\subseteq}}$ (G,T), then (H,T) λ_ε(G,T) $\mathrel{\tilde{\subseteq}}$(H,T) λ_ε(F,T) and (F,T) λ_ε(H,Z) $\mathrel{\tilde{\subseteq}}$(G,T) λ_ε (H,Z).

Proof: if (F,T) $\mathrel{\tilde{\subseteq}}$ (G,T), then (H,T) λ_ε(G,T) $\mathrel{\tilde{\subseteq}}$(H,T) λ_ε(F,T) is obvious from *Remark 1* and *Theorem 1*. Let (F,T) $\mathrel{\tilde{\subseteq}}$ (G,T), where for all $\alpha \in T$, $F(\alpha) \subseteq G(\alpha)$. Let (F,T)$\lambda_\varepsilon$(H,Z)=(Y,T∪Z), where for all $\alpha \in T \cup Z$,

$$Y(\alpha) = \begin{cases} F(\alpha). & \alpha \in T - Z, \\ H(\alpha). & \alpha \in Z - T, \\ F(\alpha) \cup H'(\alpha). & \alpha \in T \cap Z, \end{cases}$$

Let (G,T) λ_ε (H,Z)=(W,T∪Z), where for all $\alpha \in T \cup Z$,

$$W(\alpha) = \begin{cases} G(\alpha). & \alpha \in T - Z, \\ H(\alpha). & \alpha \in Z - T, \\ G(\alpha) \cup H'(\alpha). & \alpha \in T \cap Z, \end{cases}$$

If $\alpha \in$T-Z, then $Y(\alpha)= H(\alpha)$ and $W(\alpha)=G(\alpha)$, thus $Y(\alpha)= F(\alpha) \subseteq G(\alpha) = W(\alpha)$. If $\alpha \in$T-Z, then $Y(\alpha)= H(\alpha)$ and $W(\alpha)=H(\alpha)$, thus $Y(\alpha)= H(\alpha) \subseteq H(\alpha)=W(\alpha)$. If $\alpha \in T\cap Z$, then $Y(\alpha)=F(\alpha)\cup H'(\alpha)$ and $W(\alpha)=G(\alpha)\cup H'(\alpha)$, thus $Y(\alpha)= F(\alpha) \cup H'(\alpha) \subseteq G(\alpha)\cup H'(\alpha)=W(\alpha)$. Thus, for all $\alpha \in T\cup Z$, $Y(\alpha)\subseteq W(\alpha)$. Hence,

(F,T) λ_ε(H,Z) $\mathrel{\tilde{\subseteq}}$(G,T) λ_ε(H,Z).

If (F,T)λ_ε(H,Z)$\mathrel{\tilde{\subseteq}}(G,T)\lambda_\varepsilon$(H,Z), then (F,T)$\mathrel{\tilde{\subseteq}}$ (G,T) needs not be true. Similarly, if (H,T) λ_ε(G,T) $\mathrel{\tilde{\subseteq}}$(H,T) λ_ε(F,T), then (F,T) $\mathrel{\tilde{\subseteq}}$ (G,T) needs not be true.

Proof: let $E=\{e_1,e_2,e_3,e_4,e_5.e_6\}$ be the parameter set, $T=\{e_1,e_3\}$ and $Z=\{e_1,e_3.e_5\}$ be the subsets of E, $U=\{h_1,h_2.h_3,h_4.h_5\}$ be the universal set, and (F,T), (G,T) and (H,Z) be soft sets over U such that $(F,T)=\{(e_1.\{h_2.h_5\}), (e_3,\{h_1,h_2,h_5\})\}$, $(G,T)=\{(e_1,\{h_2\}), (e_3,\{h_1,h_2\})\}$, $(H,Z)=\{(e_1,\emptyset),(e_3,\emptyset),(e_5,\{h_2\})\}$. Let (F,T)$\lambda_\varepsilon$(H,Z)=(L,T∪ Z), where for all $\alpha \in T \cup Z=\{e_1.e_3.e_5\}$, $L(e_1)=H(e_1)\cup F'(e_1)=U$, $L(e_3)=H(e_3)\cup F'(e_3)=U$ and $L(e_5)=H(e_5)=\{h_2\}$. Thus, (F,T)λ_ε(H,Z)=$\{(e_1,U), (e_3,U), (e_5, \{h_2\})\}$. Now let (G,T)$\lambda_R$(H,Z)=(W,T ∪ Z), where for all $\alpha \in T \cup Z=\{e_1.e_3.e_5\}$, $W(e_1)=H(e_1)\cup G'(e_1)=U$, $W(e_3)=H(e_3)\cup G'(e_3)=U$ and $W(e_5)=H(e_5)=\{h_2\}$. Hence, (G,T) λ_ε (H,Z) =$\{(e_1, U), (e_3,U), (e_5, \{h_2\})\}$.

Thus, it is observed that (F,T) λ_ε(H,Z)$\mathrel{\tilde{\subseteq}}$(G,T) λ_ε(H,Z), but (F,T) is not a soft subset of $(G.T)$. Similarly, if (H,T) λ_ε (G,T) $\mathrel{\tilde{\subseteq}}$ (H,T)λ_ε(F,T), then (F,T) $\mathrel{\tilde{\subseteq}}$ $(G.T)$ needs not be true can be shown by choosing $(H,T)=\{(e_1,U),(e_3,U)\}$ in the above example.

If (F,T) $\mathrel{\tilde{\subseteq}}$ $(G.T)$ and (K,T) $\mathrel{\tilde{\subseteq}}$ $(L.T)$, then (F,T) λ_ε (L,T) $\mathrel{\tilde{\subseteq}}$ (G,T) λ_ε (K,T) and (K,T) λ_ε (G,T) $\mathrel{\tilde{\subseteq}}$ (L,T) λ_ε (F,T).

Proof: the proof follows from *Remark 1* and *Theorem 1*.

Theorem 7. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, extended lambda operation distributes over other soft set operations as follows:

Theorem 8. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, extended lambda operation distributes over restricted soft set operations as follows:

I. LHS distributions

If T∩(ZΔM)=∅, then (F,T) λ_ε [(G,Z)$\cap_R$(H,M)]=[(F,T) λ_ε(G,Z)] $\cap_R$ [(F,T) λ_ε (H,M)].

Proof: consider first the LHS. Let (G,Z) $\cap_R$(H,M)=(R,Z∩M), where for all α∈Z∩M, R(α)=G(α)∩H(α). Let (F,T) λ_ε(R,Z∩M)=(L,T∪(Z∩M)), where for all α∈T∪(Z∩M),

$$L(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-(Z\cap M),\\ R(\alpha). & \alpha\in (Z\cap M)-T,\\ F(\alpha)\cup R'(\alpha). & \alpha\in T\cap(Z\cap M),\end{cases}$$

thus,

$$L(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-(Z\cap M),\\ G(\alpha)\cap H(\alpha). & \alpha\in (Z\cap M)-T,\\ F(\alpha)\cup[G'(\alpha)\cup H'(\alpha)]. & \alpha\in T\cap(Z\cap M),\end{cases}$$

Now consider the RHS, i.e. [(F,T) λ_ε(G,Z)]$\cap_R$[(F,T) λ_ε (H,M)]. (F,T)λ_ε(G,Z)=(M,T∪Z), where for all α∈T∪Z,

$$M(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Let (F,T) λ_ε (H,M)=(K,T∪M), where for all α∈T∪M,

$$K(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-M,\\ H(\alpha). & \alpha\in M-T,\\ F(\alpha)\cup H'(\alpha). & \alpha\in T\cap M,\end{cases}$$

Assume that (M,T∪Z)$\cap_R$(K,T∪M)=(W,(T∪Z)∩(T∪M)), where for all α∈(T∪Z)∩(T∪M), W(α)=T(α)∩K(α). Thus,

$$W(\alpha)=\begin{cases}F(\alpha)\cap F(\alpha). & \alpha\in (T-Z)\cap(T-M)=T\cap Z'\cap M',\\ F(\alpha)\cap H(\alpha). & \alpha\in (T-Z)\cap(M-T)=\emptyset,\\ F(\alpha)\cap[F(\alpha)\cup H'(\alpha)]. & \alpha\in (T-Z)\cap(T\cap M)=\ T\cap Z'\cap M,\\ G(\alpha)\cap F(\alpha). & \alpha\in (Z-T)\cap(T-M)=\emptyset,\\ G(\alpha)\cap H(\alpha). & \alpha\in (Z-T)\cap(M-T)=T'\cap Z\cap M,\\ G(\alpha)\cap[F(\alpha)\cup H'(\alpha)]. & \alpha\in (Z-T)\cap(T\cap M)=\ \emptyset,\\ [F(\alpha)\cup G'(\alpha)]\cap F(\alpha). & \alpha\in (T\cap Z)\cap(T-M)=T\cap Z\cap M',\\ [F(\alpha)\cup G'(\alpha)]\cap H(\alpha). & \alpha\in (T\cap Z)\cap(M-T)=\ \emptyset,\\ [F(\alpha)\cup G'(\alpha)]\cap[F(\alpha)\cup H'(\alpha)]. & \alpha\in (T\cap Z)\cap(T\cap M)=T\cap Z\cap M,\end{cases}$$

Hence,

$$W(\alpha)=\begin{cases}F(\alpha). & \alpha\in T\cap Z'\cap M',\\ F(\alpha). & \alpha\in T\cap Z'\cap M,\\ G(\alpha)\cap H(\alpha). & \alpha\in T'\cap Z\cap M,\\ F(\alpha). & \alpha\in T\cap Z\cap M',\\ F'(\alpha)\cup[G'(\alpha)\cap H'(\alpha)]. & \alpha\in T\cap Z\cap M,\end{cases}$$

When considering the T-(Z∩M) in the function N, since T-(Z∩M)=T-(Z∩M)', if an element is in the complement of (Z∩M), then it is either in Z-M, or M-Z, or (Z∪M)'. Thus, if α∈ T-(Z∩M), then α∈ T∩Z∩M' or α∈ T∩Z'∩M or α∈ T∩Z'∩M'. Therefore, L=W under the condition T∩Z'∩M= T∩Z∩M'=∅, that is T∩(ZΔM)=∅.

Here, if Z∩M=∅ and T∩(ZΔM)=∅. Then N(α)=W(α)=F(α), thus N is equal to W again. Similarly, if (T∪Z)∩(T∪M)=T∪(Z∩M)=∅, that is T=∅ and Z∩M=∅, then (N,T∪(Z∩M))=(W,(T∪Z)∩(T∪M))=$\emptyset_\emptyset$. That is, in the theorem, there is no condition that the intersection of the parameter sets of the soft sets whose restricted difference will be calculated must be different from empty.

If T∩(ZΔM)= T∩Z∩M=∅, then (F,T) λ_ε [(G,Z)$\cup_R$(H,M)]= [(F,T) λ_ε(G,Z)] $\cup_R$ [(F,T) λ_ε(H,M)].

II. RHS Distributions

If (TΔZ)∩M=∅, then [(F,T) $\cap_R$(G,Z)]λ_ε(H,M)= [(F,T)λ_ε(H,M)] $\cap_R$ [(G,Z) λ_ε (H,M)].

Proof: consider first the LHS. Let (F,T)$\cap_R$(G,Z)=(R,T∩Z, where for all α∈T∩Z, R(α)=F(α)∩G(α). Let (R,T∩Z) λ_ε(H,M)=(L,(T∩Z)∪M), where for all α∈(T∩Z)∪M,

$$L(\alpha)=\begin{cases} R(\alpha). & \alpha\in(T\cap Z)-M,\\ H(\alpha). & \alpha\in M-(T\cap Z),\\ R(\alpha)\cup H'(\alpha). & \alpha\in(T\cap Z)\cap M,\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha)\cap G(\alpha). & \alpha\in(T\cap Z)-M,\\ H(\alpha). & \alpha\in M-(T\cap Z),\\ [F(\alpha)\cap G(\alpha)]\cup H'(\alpha). & \alpha\in(T\cap Z)\cap M,\end{cases}$$

Now consider the RHS, i.e. [(F,T)λ_ε(H,M)] $\cap_R$ [(G,Z) λ_ε (H,M)]. Let (F,T)λ_ε(H,M)=(S,T∪M), where for all α∈T∪M,

$$S(\alpha)=\begin{cases} F(\alpha). & \alpha\in T-M,\\ H(\alpha). & \alpha\in M-T,\\ F(\alpha)\cup H'(\alpha). & \alpha\in T\cap M,\end{cases}$$

Let (G,Z) λ_ε (H,M)=(K,Z∪M), where for all α∈Z∪M,

$$K(\alpha)=\begin{cases} G(\alpha). & \alpha\in Z-M,\\ H(\alpha). & \alpha\in M-Z,\\ G(\alpha)\cup H'(\alpha). & \alpha\in Z\cap M,\end{cases}$$

Assume that (S,T∪Z)$\cap_R$(K,Z∪M)=(W,(T∪Z)∩(Z∪M)), where for all α∈(T∪Z)∩(Z∪M), where for W(α)=S(α)∩K(α). Thus,

$$W(\alpha)=\begin{cases} F(\alpha)\cap G(\alpha). & \alpha\in(T-M)\cap(Z-M)=T\cap Z\cap M',\\ F(\alpha)\cap H(\alpha). & \alpha\in(T-M)\cap(M-Z)=\emptyset,\\ F(\alpha)\cap[G(\alpha)\cup H'(\alpha)]. & \alpha\in(T-M)\cap(Z\cap M)=\emptyset,\\ H(\alpha)\cap G(\alpha). & \alpha\in(M-T)\cap(Z-M)=\emptyset,\\ H(\alpha)\cap H(\alpha). & \alpha\in(M-T)\cap(M-Z)=T'\cap Z'\cap M,\\ H(\alpha)\cap[G(\alpha)\cup H'(\alpha)]. & \alpha\in(M-T)\cap(Z\cap M)=T'\cap Z\cap M,\\ [F(\alpha)\cup H'(\alpha)]\cap G(\alpha). & \alpha\in(T\cap M)\cap(Z-M)=\emptyset,\\ [F(\alpha)\cup H'(\alpha)]\cap H(\alpha). & \alpha\in(T\cap M)\cap(M-Z)=T\cap Z'\cap M,\\ [F(\alpha)\cup H'(\alpha)]\cap[G(\alpha)\cup H'(\alpha)]. & \alpha\in(T\cap M)\cap(Z\cap M)=T\cap Z\cap M,\end{cases}$$

Therefore,

$$W(\alpha)=\begin{cases} F(\alpha)\cap G(\alpha). & \alpha\in T\cap Z\cap M',\\ H(\alpha). & \alpha\in T'\cap Z'\cap M,\\ H(\alpha)\cap G(\alpha). & \alpha\in T'\cap Z\cap M,\\ F(\alpha)\cap H(\alpha). & \alpha\in T\cap Z'\cap M,\\ [F(\alpha)\cap G(\alpha)]\cup H'(\alpha). & \alpha\in T\cap Z\cap M,\end{cases}$$

When considering M-(T∩Z) in the function L, since M-(T∩Z)=M∩(T∩Z)', if an element is in the complement of (T∩Z), then it is either in T-Z, or in Z-T or in (T∪Z)'. Thus, if α∈M-(T∩Z), then either α∈M∩T∩Z' or α∈ M∩Z∩T' or α∈ M∩T'∩Z'. Therefore, L=W under the condition T'∩Z∩M=T∩Z'∩M=∅.

Here, if T∩Z=∅, then L(α)=W(α)=H(α), thus N is equal to W again. Similarly, if (T∪M)∩(Z∪M)=(T∩Z)∪M=∅, that is T∩Z=∅ and M=∅, then (L,(T∩Z)∪M)=(W,(T∪M)∩(Z∪M))=$\emptyset_\emptyset$. That is, in the theorem, there is no condition that the intersection of the parameter sets of the soft sets whose restricted difference will be calculated must be different from empty.

If $(T\Delta Z) \cap M = \emptyset$, then $[(F.T) \cup_R (G.Z)]\ \lambda_\varepsilon(H.M) = [(F.T)\ \lambda_\varepsilon(H.M)] \cup_R [(G.Z)\ \lambda_\varepsilon\ (H.M)]$.

Corollary 2. $(S_E(U).\cap_R.\lambda_\varepsilon)$ is an additive idempotent noncommutative (right) nearsemiring with unity and zero but without zero-symmetric properties and under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U).\cap_R)$ is a commutative, idempotent monoid with identity U_E, that is, a bounded semilattice (hence a semigroup). By Corollary 1, $(S_E(U).\lambda_\varepsilon)$ is a noncommutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition $T \cap Z \cap M = \emptyset$, where $(F.T)$, $(G.Z)$ and $(H.M)$ are soft sets over U. Moreover, by Theorem 8, λ_ε distributes over $\cap_R$ from LHS under $T \cap (Z\Delta M) = \emptyset$ and by Theorem 6, $U_E\lambda_\varepsilon(F.A) = U_E$ that is, U_E is left absorbing element for the operation λ_ε in $S_E(U)$. Thus, $(S_E(U).\cap_R.\lambda_\varepsilon)$ is an additive idempotent noncommutative (right) nearsemiring with unity and zero under the condition $T \cap Z \cap M = T \cap (Z\Delta M) = \emptyset$. Moreover, since $(F.A)\lambda_\varepsilon U_E \neq U_E$, $(S_E(U).\cap_R.\lambda_\varepsilon)$ is a (right) nearsemiring without zero-symmetric property and under certain conditions.

Corollary 3. $(S_E(U),\cap_R,\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U),\cap_R)$ is a commutative, idempotent monoid with identity U_E, that is, a bounded semilattice (hence a semigroup). By *Corollary 1*, $(S_E(U),\lambda_\varepsilon)$ is a noncommutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 8*, λ_ε distributes over $\cap_R$ from LHS under T∩(ZΔM)= T∩Z∩M=∅ and by *Theorem 8*, λ_ε distributes over $\cap_R$ form RHS under the condition (TΔZ)∩M=∅. Consequently, under the condition T∩Z∩M=(TΔZ)∩M=T∩(ZΔM)=∅, $(S_E(U),\cap_R,\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Corollary 4. $(S_E(U),\cup_R,\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U),\cup_R)$ is a commutative, idempotent monoid with identity $\emptyset_\emptyset$, that is, a bounded semilattice (hence a semigroup). By *Corollary 1*, $(S_E(U).\lambda_\varepsilon)$ is a noncommutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 8*, λ_ε distributes over $\cup_R$ from LHS under T∩(ZΔM)= T∩Z∩M=∅ and by *Theorem 8*, λ_ε distributes over $\cup_R$ form RHS under the condition (TΔZ)∩M=∅. Consequently, under the condition T∩Z∩M=T∩(ZΔM)=(TΔZ)∩M=∅, $(S_E(U),\cup_R,\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Theorem 9. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, the extended lambda operation distributes over other extended soft set operations as follows:

I. LHS distributions

If T∩(ZΔM) = T∩Z∩M=∅, then (F,T) λ_ε[(G,Z) $\cap_\varepsilon$ (H,M)]=[(F,T)λ_ε(G,Z)] $\cap_\varepsilon$[(F,T) λ_ε (H,M)].

Proof: first, consider the LHS. Let (G,Z) $\cap_\varepsilon$(H,M)=(R,Z∪M), where for all α∈Z∪M,

$$M(\alpha) = \begin{cases} G(\alpha). & \alpha \in Z - M, \\ H(\alpha). & \alpha \in M - Z, \\ G(\alpha) \cap H(\alpha). & \alpha \in Z \cap M, \end{cases}$$

Thus,

$$N(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-(Z\cup M)=T\cap Z'\cap M',\\ G(\alpha). & \alpha\in(Z-M)-T=T'\cap Z\cap M',\\ H(\alpha). & \alpha\in(M-Z)-T=T'\cap Z'\cap M,\\ G(\alpha)\cap H(\alpha). & \alpha\in(Z\cap M)-T=T'\cap Z\cap M,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap(Z-M)=T\cap Z\cap M',\\ F(\alpha)\cup H'(\alpha). & \alpha\in T\cap(M-Z)=T\cap Z'\cap M,\\ F(\alpha)\cup[G'(\alpha)\cup H'(\alpha)]. & \alpha\in T\cap(Z\cap M)=T\cap Z\cap M,\end{cases}$$

Now consider the RHS i.e. $[(F,T)\lambda_\varepsilon(G,Z)]\cap_\varepsilon[(F,T)\lambda_\varepsilon(H,M$. Let $(F,T)\lambda_\varepsilon(G,Z)=(K,T\cup Z)$ where for all $\alpha\in T\cup Z$,

$$K(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Let $(F,T)\lambda_\varepsilon(H,M)=(S,T\cup M)$, where for all $\alpha\in T\cup M$,

$$S(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-M,\\ H(\alpha). & \alpha\in M-T,\\ F(\alpha)\cup H'(\alpha). & \alpha\in T\cap M,\end{cases}$$

Let $(K,T\cup Z)\cap_\varepsilon(S,T\cup M)=(L,(T\cup Z)\cup(T\cup M))$, where for all $\alpha\in(T\cup Z)\cup(T\cup M)$,

$$L(\alpha)=\begin{cases}K(\alpha). & \alpha\in(T\cup Z)-(T\cup M),\\ S(\alpha). & \alpha\in(T\cup M)-(T\cup Z),\\ K(\alpha)\cap S(\alpha). & \alpha\in(T\cup Z)\cap(T\cup M),\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases}K(\alpha). & \alpha\in(T\cup Z)-(T\cup M),\\ S(\alpha). & \alpha\in(T\cup M)-(T\cup Z),\\ K(\alpha)\cap S(\alpha). & \alpha\in(T\cup Z)\cap(T\cup M),\end{cases}$$

$$L(\alpha)=\begin{cases}F(\alpha). & \alpha\in(T-Z)-(T\cup M)=\emptyset,\\ G(\alpha). & \alpha\in(Z-T)-(T\cup M)=T'\cap Z\cap M',\\ F(\alpha)\cup G'(\alpha). & \alpha\in(T\cap Z)-(T\cup M)=\emptyset,\\ F(\alpha). & \alpha\in(T-M)-(T\cup Z)=\emptyset,\\ H(\alpha). & \alpha\in(M-T)-(T\cup Z)=T'\cap Z'\cap M,\\ F(\alpha)\cup H'(\alpha). & \alpha\in(T\cap M)-(T\cup Z)=\emptyset,\\ F(\alpha)\cap F(\alpha). & \alpha\in(T-Z)\cap(T-M)=T\cap Z'\cap M',\\ F(\alpha)\cap H(\alpha). & \alpha\in(T-Z)\cap(M-T)=\emptyset,\\ F(\alpha)\cap[F(\alpha)\cup H'(\alpha)]. & \alpha\in(T-Z)\cap(T\cap M)=T\cap Z'\cap M,\\ G(\alpha)\cap F(\alpha). & \alpha\in(Z-T)\cap(T-M)=\emptyset,\\ G(\alpha)\cap H(\alpha). & \alpha\in(Z-T)\cap(M-T)=T'\cap Z\cap M,\\ G(\alpha)\cap[F(\alpha)\cup H'(\alpha)]. & \alpha\in(Z-T)\cap(T\cap M)=\emptyset,\\ [F(\alpha)\cup G'(\alpha)]\cap F(\alpha). & \alpha\in(T\cap Z)\cap(T-M)=T\cap Z\cap M',\\ [F(\alpha)\cup G'(\alpha)]\cap H(\alpha). & \alpha\in(T\cap Z)\cap(M-T)=\emptyset,\\ [F(\alpha)\cup G'(\alpha)]\cap[F(\alpha)\cup H'(\alpha)]. & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap Z\cap M,\end{cases}$$

Hence,

$$L(\alpha)=\begin{cases}G(\alpha). & \alpha\in T'\cap Z\cap M',\\ H(\alpha). & \alpha\in T'\cap Z'\cap M,\\ F(\alpha). & \alpha\in T\cap Z'\cap M',\\ F(\alpha). & \alpha\in T\cap Z'\cap M,\\ G(\alpha)\cap H(\alpha). & \alpha\in T'\cap Z\cap M,\\ F(\alpha). & \alpha\in T\cap Z\cap M',\\ F(\alpha)\cup[G'(\alpha)\cap H'(\alpha)]. & \alpha\in T\cap Z\cap M,\end{cases}$$

Hence, N=L, where $T\cap Z\cap M'=T\cap Z'\cap M=T\cap Z\cap M=\emptyset$. It is obvious that the condition $T\cap Z\cap M'=T\cap Z'\cap M=\emptyset=T\cap Z\cap M=\emptyset$ is equal to the condition $T\cap(Z\Delta M)=\emptyset$.

If $T\cap Z\cap M=\emptyset$, then $(F,T)\ \lambda_\varepsilon[(G,Z)\cup_\varepsilon(H,M)]=[(F,T)\lambda_\varepsilon(G,Z)]\cup_\varepsilon[(F,T)\ \lambda_\varepsilon\ (H,M)]$

II. RHS distributions

If $(T\Delta Z)\cap M=\emptyset$, then $[(F,T)\cup_\varepsilon(G,Z)]\ \lambda_\varepsilon(H,M)=[(F,T)\ \lambda_\varepsilon(H,M)]\cup_\varepsilon[(G,Z)\ \lambda_\varepsilon(H,M)]$.

Proof: first, consider the LHS. Let $(F,T)\cup_\varepsilon(G,Z)=(R,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$R(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Let $(R,T\cup Z)\ \lambda_\varepsilon\ (H,M)=(N,(T\cup Z)\cup M)$, where for all $\alpha\in(T\cup Z)\cup M$,

$$N(\alpha)=\begin{cases}R(\alpha). & \alpha\in(T\cup Z)-M,\\ H(\alpha). & \alpha\in M-(T\cup Z),\\ R(\alpha)\cup H'(\alpha). & \alpha\in(T\cup Z)\cap M,\end{cases}$$

Thus,

$$N(\alpha)=\begin{cases}F(\alpha). & \alpha\in(T-Z)-M=T\cap Z'\cap M',\\ G(\alpha). & \alpha\in(Z-T)-M=T'\cap Z\cap M',\\ F(\alpha)\cup G(\alpha). & \alpha\in(T\cap Z)-M=T\cap Z\cap M',\\ H(\alpha). & \alpha\in M-(T\cup Z)=T'\cap Z'\cap M,\\ F(\alpha)\cup H'(\alpha). & \alpha\in(T-Z)\cap M=T\cap Z'\cap M,\\ G(\alpha)\cup H'(\alpha). & \alpha\in(Z-T)\cap M=T'\cap Z\cap M,\\ [F(\alpha)\cup G(\alpha)]\cup H'(\alpha). & \alpha\in(T\cap Z)\cap M=T\cap Z\cap M,\end{cases}$$

Now consider the RHS, i.e. $[(F,T)\ \lambda_\varepsilon(H,M)]\ \cup_\varepsilon[(G,Z)\ \lambda_\varepsilon(H,M)]$. Let $(F,T)\ \lambda_\varepsilon(H,M)=(K,T\cup Z)$, where for all $\alpha\in T\cup M$,

$$K(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-M,\\ H(\alpha). & \alpha\in M-T,\\ F(\alpha)\cup H'(\alpha). & \alpha\in T\cap M,\end{cases}$$

Let $(G,Z)\ \lambda_\varepsilon\ (H,M)=(S,T\cup M)$, where for all $\alpha\in Z\cup M$,

$$S(\alpha)=\begin{cases}G(\alpha). & \alpha\in Z-M,\\ H(\alpha). & \alpha\in M-Z,\\ G(\alpha)\cup H'(\alpha). & \alpha\in Z\cap M,\end{cases}$$

Assume that $(K,T\cup M)\ \cup_\varepsilon(S,Z\cup M)=(L,(T\cup M)\cup(Z\cup M))$, where for all $\alpha\in(T\cup M)\cup(Z\cup M)$,

$$L(\alpha)=\begin{cases}K(\alpha). & \alpha\in(T\cup M)-(Z\cup M),\\ S(\alpha). & \alpha\in(Z\cup M)-(T\cup M),\\ K(\alpha)\cup S(\alpha). & \alpha\in(T\cup M)\cap(Z\cup M),\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases}F(\alpha). & \alpha\in(T-M)-(Z\cup M)=T\cap Z'\cap M',\\ H(\alpha). & \alpha\in(M-T)-(Z\cup M)=\emptyset,\\ F(\alpha)\cup H'(\alpha). & \alpha\in(T\cap M)-(Z\cup M)=\emptyset,\\ L(\alpha)=G(\alpha). & \alpha\in(Z-M)-(T\cup M)=T'\cap Z\cap M',\\ H(\alpha). & \alpha\in(M-Z)-(T\cup M)=\emptyset,\\ G(\alpha)\cup H'(\alpha). & \alpha\in(Z\cap M)-(T\cup M)=\emptyset,\\ F(\alpha)\cup G(\alpha). & \alpha\in(T-M)\cap(Z-M)=T\cap Z\cap M',\\ F(\alpha)\cup H(\alpha). & \alpha\in(T-M)\cap(M-Z)=\emptyset,\\ F(\alpha)\cup[G(\alpha)\cup H'(\alpha)]. & \alpha\in(T-M)\cap(Z\cap M)=\emptyset,\\ H(\alpha)\cup G(\alpha). & \alpha\in(M-T)\cap(Z-M)=\emptyset,\\ H(\alpha)\cup H(\alpha). & \alpha\in(M-T)\cap(M-Z)=T'\cap Z'\cap M,\\ H(\alpha)\cup[G(\alpha)\cup H'(\alpha)]. & \alpha\in(M-T)\cap(Z\cap M)=T'\cap Z\cap M,\\ [F(\alpha)\cup H'(\alpha)]\cup G(\alpha). & \alpha\in(T\cap M)\cap(Z-M)=\emptyset,\\ [F(\alpha)\cup H'(\alpha)]\cup H(\alpha). & \alpha\in(T\cap M)\cap(M-Z)=T\cap Z'\cap M,\\ [F(\alpha)\cup H'(\alpha)]\cup[G(\alpha)\cup H'(\alpha)]. & \alpha\in(T\cap M)\cap(Z\cap M)=T\cap Z\cap M,\end{cases}$$

Hence,

$$L(\alpha)=\begin{cases}F(\alpha). & \alpha\in T\cap Z'\cap M',\\ G(\alpha). & \alpha\in T'\cap Z\cap M',\\ F(\alpha)\cup G(\alpha). & \alpha\in T\cap Z\cap M',\\ H(\alpha). & \alpha\in T'\cap Z'\cap M,\\ U. & \alpha\in T'\cap Z\cap M,\\ U. & \alpha\in T\cap Z'\cap M,\\ F(\alpha)\cup G(\alpha)\cup H'(\alpha). & \alpha\in T\cap Z\cap M,\end{cases}$$

Therefore, N=L, where $T'\cap Z\cap M=T\cap Z'\cap M=\emptyset$. It is obvious that the condition $T'\cap Z\cap M=\emptyset=T\cap Z'\cap M=\emptyset$ is equal to the condition $(T\Delta Z)\cap M=\emptyset$.

If $(T\Delta Z)\cap M=\emptyset$, then $[(F,T)\cap_\varepsilon(G,Z)]\lambda_\varepsilon(H,M)=[(F,T)\lambda_\varepsilon(H,M)]\cap_\varepsilon[(G,Z)\lambda_\varepsilon(H,M)]$.

Corollary 5. $(S_E(U),\cup_\varepsilon,\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U).\cup_\varepsilon)$ is a commutative, idempotent monoid with identity $\emptyset_\emptyset$, that is, a bounded semilattice (hence a semigroup). By *Corollary 1*, $(S_E(U),\lambda_\varepsilon)$ is a noncommutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition $T\cap Z\cap M=\emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 9*, λ_ε distributes over $\cup_\varepsilon$ from LHS under $T\cap Z\cap M=\emptyset$, and by *Theorem 9*, λ_ε distributes over $\cup_\varepsilon$ from RHS under the condition $(T\Delta Z)\cap M=\emptyset$. Consequently, under the condition $T\cap Z\cap M=T\cap(Z\Delta M)=\emptyset$, $(S_E(U),\cup_\varepsilon.\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Corollary 7. $(S_E(U),\cap_\varepsilon,\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U).\cap_\varepsilon)$ is a commutative, idempotent monoid with identity $\emptyset_\emptyset$, that is, a bounded semilattice (hence a semigroup). By *Corollary 1*, $(S_E(U).\lambda_\varepsilon)$ is a noncommutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition $T\cap Z\cap M=\emptyset$, where $(F.T)$, $(G.Z)$ and (H,M) are soft sets over U. Moreover, by *Theorem 9*, λ_ε distributes over $\cap_\varepsilon$ from LHS under $T\cap(Z\Delta M)=T\cap Z\cap M=\emptyset$, and by

Theorem 9, λ_ε distributes over $\cap_\varepsilon$ from RHS under the condition $(T\Delta Z)\cap M=\emptyset$. Consequently, under the condition $T\cap Z\cap M=T\cap(Z\Delta M)=(T\Delta Z)\cap M=\emptyset$ $(S_E(U),\cap_\varepsilon,\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Theorem 10. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, extended lambda operation distributes over soft binary piecewise operations as follows:

I. LHS Distributions

If $T\cap Z\cap M'=T\cap Z\cap M=\emptyset$, then $(F,T)\lambda_\varepsilon[(G,Z)\,\tilde{\cap}\,(H,M)] = [(F,T)\lambda_\varepsilon(G,Z)]\,\tilde{\cap}\,[(F,T)\,\lambda_\varepsilon\,(H,M)]$.

Proof: first, consider the LHS. Let $(G,Z)\,\tilde{\cap}\,(H,M)=(R,Z)$, where for all $\alpha\in Z$,

$$R(\alpha)=\begin{cases}G(\alpha). & \alpha\in Z-M,\\ G(\alpha)\cap H(\alpha). & \alpha\in Z\cap M,\end{cases}$$

$(F,T)\,\lambda_\varepsilon\,(R,Z)=(N,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$N(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ R(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup R'(\alpha). & \alpha\in T\cap Z,\end{cases}$$

Thus,

$$N(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in (Z-M)-T=T'\cap Z\cap M',\\ G(\alpha)\cap H(\alpha). & \alpha\in (Z\cap M)-T=T'\cap Z\cap M,\\ F(\alpha)\cup G'(\alpha). & \alpha\in T\cap(Z-M)=T\cap Z\cap M',\\ F(\alpha)\cup[G'(\alpha)\cup H'(\alpha)]. & \alpha\in T\cap(Z\cap M)=T\cap Z\cap M.\end{cases}$$

Now consider the RHS, i.e. $[(F,T)\lambda_\varepsilon(G,Z)]\,\tilde{\cap}\,[(F,T)\lambda_\varepsilon(H,M)]$. Let $(F,T)\lambda_\varepsilon(G,Z)=(K,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$K(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-Z,\\ G(\alpha). & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha), & \alpha\in T\cap Z,\end{cases}$$

Let $(F,T)\,\lambda_\varepsilon\,(H,M)=(S,T\cup M)$, where for all $\alpha\in T\cup M$,

$$S(\alpha)=\begin{cases}F(\alpha). & \alpha\in T-M,\\ H(\alpha). & \alpha\in M-T,\\ F(\alpha)\cup H'(\alpha). & \alpha\in T\cap M,\end{cases}$$

Let $(K,T\cup Z)\,\tilde{\cap}\,(S,T\cup M)=(L,(T\cup Z)\cup(T\cup M))$, where for all $\alpha\in(T\cup Z)\cup(T\cup M)$,

$$L(\alpha)=\begin{cases}K(\alpha). & \alpha\in (T\cup Z)-(T\cup M),\\ K(\alpha)\cap S(\alpha). & \alpha\in (T\cup Z)\cap(T\cup M),\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha). & \alpha\in(T-Z)-(T\cup M)=\emptyset,\\ G(\alpha). & \alpha\in(Z-T)-(T\cup M)=T'\cap Z\cap M',\\ F(\alpha)\cup G'(\alpha). & \alpha\in(T\cap Z)-(T\cup M)=\emptyset,\\ F(\alpha)\cap F(\alpha). & \alpha\in(T-Z)\cap(T-M)=T\cap Z'\cap M',\\ F(\alpha)\cap H(\alpha). & \alpha\in(T-Z)\cap(M-T)=\emptyset,\\ F(\alpha)\cap [F(\alpha)\cup H'(\alpha)]. & \alpha\in(T-Z)\cap(T\cap M)=T\cap Z'\cap M,\\ G(\alpha)\cap F(\alpha). & \alpha\in(Z-T)\cap(T-M)=\emptyset,\\ G(\alpha)\cap H(\alpha). & \alpha\in(Z-T)\cap(M-T)= T'\cap Z\cap M,\\ G(\alpha)\cap [F(\alpha)\cup H'(\alpha)]. & \alpha\in(Z-T)\cap(T\cap M)=\emptyset,\\ [F(\alpha)\cup G'(\alpha)]\cap F(\alpha). & \alpha\in(T\cap Z)\cap(T-M)=T\cap Z\cap M,\\ [F(\alpha)\cup G'(\alpha)]\cap H(\alpha). & \alpha\in(T\cap Z)\cap(M-T)=\emptyset,\\ [F(\alpha)\cup G'(\alpha)]\cap [F(\alpha)\cup H'(\alpha)]. & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap Z\cap M, \end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} G(\alpha). & \alpha\in T'\cap Z\cap M',\\ F(\alpha). & \alpha\in T\cap Z'\cap M',\\ F(\alpha). & \alpha\in T\cap Z'\cap M,\\ G(\alpha)\cap H(\alpha). & \alpha\in T'\cap Z\cap M,\\ F(\alpha). & \alpha\in T\cap Z\cap M',\\ F(\alpha)\cup [G'(\alpha)\cap H'(\alpha)]. & \alpha\in T\cap Z\cap M, \end{cases}$$

When considering T-Z in the function N, since T-Z=T∩Z', if an element is in the complement of Z, it is either in M-Z, or (M∪Z)'. Thus, if α∈T-Z, then either α∈T∩M∩Z' or α∈T∩M'∩Z', hence N=L where T∩Z∩M'=T∩Z∩M=∅.

If T∩Z∩M=∅, then $(F,T)\ \lambda_\varepsilon\ [(G,Z)\ \tilde{\cup}\ (H,M)] = [(F,T)\ \lambda_\varepsilon\ (G,Z)]\ \tilde{\cup}\ [(F,M)\ \lambda_\varepsilon\ (H,M)]$.

II. RHS Distributions

If T'∩Z∩M= T∩Z∩M =∅, $[(F,T)\tilde{\cup}\ (G,Z)]\ \lambda_\varepsilon\ (H,M) = [(F,T)\ \lambda_\varepsilon(H,M)]\ \tilde{\cup}\ [(G,Z)\ \tilde{\lambda}(H,M)]$.

Proof: first, consider the LHS of the equality. Let $(F,T)\ \tilde{\cup}\ (G,Z)=(R,T)$, where for all α∈T,

$$R(\alpha)=\begin{cases} F(\alpha). & \alpha\in T-Z,\\ F(\alpha)\cup G(\alpha). & \alpha\in T\cap Z, \end{cases}$$

Let $(R,T)\ \lambda_\varepsilon\ (H,M) =(N,T\cup M)$, where for all α∈T∪M,

$$N(\alpha)=\begin{cases} R(\alpha). & \alpha\in T-M,\\ H(\alpha). & \alpha\in M-T,\\ R(\alpha)\cup H'(\alpha). & \alpha\in T\cap M, \end{cases}$$

Thus,

$$N(\alpha)=\begin{cases} F(\alpha). & \alpha\in(T-Z)-M=T\cap Z'\cap M',\\ F(\alpha)\cup G(\alpha). & \alpha\in(T\cap Z)-M=T\cap Z\cap M',\\ H(\alpha). & \alpha\in M-T.\\ F(\alpha)\cup H'(\alpha). & \alpha\in(T-Z)\cap M=T\cap Z'\cap M,\\ [F(\alpha)\cup G(\alpha)]\cup H'(\alpha). & \alpha\in(T\cap Z)\cap M=T\cap Z\cap M, \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\ \lambda_\varepsilon(H,M)]\ \tilde{\cup}\ [(G,Z)\ \tilde{\lambda}(H,M)]$. Let $(F,T)\ \lambda_\varepsilon\ (H,M)=(K,T\cup M)$, where for all α∈T∪M,

$$K(\alpha) = \begin{cases} F(\alpha). & \alpha \in T - M, \\ H(\alpha). & \alpha \in M - T, \\ F(\alpha) \cup H'(\alpha). & \alpha \in T \cap M, \end{cases}$$

Let (G,Z) λ_ε (H,M)=(S,T∪M), where for all α∈Z∪M,

$$S(\alpha) = \begin{cases} G(\alpha). & \alpha \in Z - M, \\ H(\alpha). & \alpha \in M - Z, \\ G(\alpha) \cup H'(\alpha). & \alpha \in Z \cap M, \end{cases}$$

Let (K,T∪M) $\tilde{\cup}$ (S,Z∪M)=(L,(T∪M)∪(Z∪M)), where for all α∈(T∪M)∪(Z∪M),

$$L(\alpha) = \begin{cases} K(\alpha). & \alpha \in (T \cup M) - (Z \cup M), \\ K(\alpha) \cup S(\alpha). & \alpha \in (T \cup M) \cap (Z \cup M), \end{cases}$$

Thus,

$$L(\alpha) = \begin{cases} F(\alpha). & \alpha \in (T - M) - Z = T \cap Z' \cap M', \\ H(\alpha). & \alpha \in (M - T) - Z = T' \cap Z' \cap M, \\ F(\alpha) \cup H'(\alpha). & \alpha \in (T \cap M) - Z = T \cap Z' \cap M, \\ F(\alpha) \cup G(\alpha). & \alpha \in (T - M) \cap (Z - M) = T \cap Z \cap M', \\ F(\alpha) \cup [G(\alpha) \cup H'(\alpha)]. & \alpha \in (T - M) \cap (Z \cap M) = \emptyset, \\ H(\alpha) \cup G(\alpha). & \alpha \in (M - T) \cap (Z - M) = \emptyset, \\ H(\alpha) \cup [G(\alpha) \cup H'(\alpha)]. & \alpha \in (M - T) \cap (Z \cap M) = T' \cap Z \cap M, \\ [F(\alpha) \cup H'(\alpha)] \cup G(\alpha). & \alpha \in (T \cap M) \cap (Z - M) = \emptyset, \\ [F(\alpha) \cup H'(\alpha)] \cup [G(\alpha) \cup H'(\alpha)]. & \alpha \in (T \cap M) \cap (Z \cap M) = T \cap Z \cap M, \end{cases}$$

Hence,

$$L(\alpha) = \begin{cases} F(\alpha). & \alpha \in T \cap Z' \cap M', \\ F(\alpha) \cup G(\alpha). & \alpha \in T \cap Z \cap M', \\ H(\alpha). & \alpha \in T' \cap Z' \cap M, \\ U. & \alpha \in T' \cap Z \cap M, \\ F(\alpha) \cup H'(\alpha). & \alpha \in T \cap Z' \cap M, \\ [F'(\alpha) \cup G'(\alpha)] \cup H(\alpha). & \alpha \in T \cap Z \cap M, \end{cases}$$

When considering M-T in the function N, since M-T=M∩T', if an element is in the complement of T, then it is either in Z-T or (Z∪T)'. Thus, if α∈M-T, then α∈ M∩Z∩T' or α∈M∩Z'∩T'. Thus, N=L under T'∩Z∩M= T∩Z∩M =∅.

If T'∩Z∩M=∅, then [(F,T) $\tilde{\cap}$ (G,Z)] λ_ε (H,M) = [(F,T) λ_ε(H,M)] $\tilde{\cap}$ [(G,Z) $\tilde{\lambda}$ (H,M)].

Corollary 8. $(S_E(U),\tilde{\cup},\lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Proof: Yavuz [31] showed that $(S_E(U), \tilde{\cup})$ is an idempotent, noncommutative semigroup (that is a band) under the condition T∩Z'∩M =∅, where (F,T), (G,Z) and (H,M) are soft sets.

By *Corollary 1*, $(S_E(U), \lambda_\varepsilon)$ is a noncommutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 10*, λ_ε distributes over $\tilde{\cup}$ from LHS under T∩Z∩M=∅, and by *Theorem 10*, λ_ε distributes over $\tilde{\cup}$ from RHS under the condition T'∩Z∩M= T∩Z∩M =∅. Consequently, under the condition T∩Z∩M=(TΔZ)∩M=∅, $(S_E(U),\tilde{\cup}, \lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Corollary 9. $(S_E(U), \tilde{\cap}, \lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Proof: Yavuz [31] showed that $(S_E(U), \tilde{\cap})$ is an idempotent, noncommutative semigroup (that is a band) under the condition $T \cap Z' \cap M = \emptyset$, where (F,T), (G,Z) and (H,M) are soft sets.

By *Corollary 1*, $(S_E(U).\lambda_\varepsilon)$ is a noncommutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition $T \cap Z \cap M = \emptyset$, where $(F.T)$, $(G.Z)$, and $(H.M)$ are soft sets over U. Moreover, by *Theorem 9*, λ_ε distributes over $\tilde{\cap}$ from LHS under $T \cap Z \cap M' = T \cap Z \cap M = \emptyset$, and by *Theorem 9*, λ_ε distributes over $\tilde{\cap}$ from RHS under the condition $T' \cap Z \cap M = \emptyset$. Consequently, under the condition $T \cap Z \cap M = T' \cap Z \cap M = T \cap (Z \Delta M) = \emptyset$, $(S_E(U), \tilde{\cap}, \lambda_\varepsilon)$ is an additive idempotent noncommutative semiring without zero but with unity under certain conditions.

Conclusion

Parametric techniques like soft sets and soft operations are extremely beneficial when dealing with uncertain things. Proposing new soft operations and deriving their algebraic features and implementations provide new insights into solving parametric data problems. In this sense, a novel kind of restricted and extended soft set operation is presented in this work. By putting forth "restricted and extended lambda operations of soft sets" and systematically examining the algebraic structures associated with these and other novel soft set operations, we want to add to the body of work on soft set theory. Specifically, these novel soft set operations' algebraic properties are analyzed in detail. Considering the algebraic properties of these soft set operations and distribution laws, an extensive analysis of the algebraic structures with these operations in the collection of soft sets over a universe is provided. We demonstrate that $(S_E(U), \lambda_\varepsilon)$ is a noncommutative monoid with identity $\emptyset_\emptyset$ under certain conditions. Furthermore, we demonstrate that extended lambda operation and other types of soft sets and operations construct several significant algebraic structures, including semirings and nearsemirings, in the collection of soft sets over the universe.

I. $(S_E(U), \cap_R, \lambda_\varepsilon)$, $(S_E(U), \cup_R, \lambda_\varepsilon)$, $(S_E(U), \cup_\varepsilon, \lambda_\varepsilon)$, $(S_E(U), \cap_\varepsilon, \lambda_\varepsilon)$, $(S_E(U), \tilde{\cap}, \lambda_\varepsilon)$, $(S_E(U), \tilde{\cup}, \lambda_\varepsilon)$ are all additive idempotent noncommutative semiring without zero but with unity under certain conditions.

II. $(S_E(U), \cap_R, \lambda_\varepsilon)$ is also additive commutative, idempotent, (right) nearsemirings with zero and unity but without a zero-symmetric property under certain conditions.

By studying novel soft set operations and the algebraic structures of soft sets, we thoroughly comprehend their use. This has the potential to advance soft set theory as well as the traditional algebraic literature in addition to providing new instances of algebraic structures. Future research might look at further varieties of new restricted and extended soft set operations and the accompanying distributions and characteristics to add to the body of knowledge.

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Author Contributions

Aslihan Sezgin conceptualized and developed the restricted and extended lambda operations, conducted theoretical analyses, and drafted the manuscript. Fitnat Nur Aybek contributed to the formulation and proof of algebraic properties and provided critical revisions. Nenad Stojanović assisted in the application of the operations to practical scenarios, including decision-making and cryptology. All authors reviewed and approved the final manuscript.

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Data Availability

This study is theoretical and does not involve experimental data. Supporting materials are available upon request from the corresponding author.

Conflicts of Interest

The authors declare no conflicts of interest related to this work.

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