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Recent Trends and Applications of Linear Programming in Network Flow: A Comprehensive Survey

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Abstract

By leveraging advanced techniques and models, Operations Research (OR) provides critical insights and strategic interventions across multiple domains, including transportation, communication, project management, and supply chain optimization. The field's multifaceted approach continues to drive efficiency and innovation in numerous industries. This paper thoroughly evaluates numerous approaches and methods researchers employ to model and investigate problems. Our objective is to bridge gaps in existing literature by examining recent advancements in this field. Network flow problems encompass the shortest path, maximal cost flow, and minimal cost flow problems. These critical elements are essential for understanding transportation dynamics, communication, and resource allocation networks. Furthermore, we explore real-life scenarios where these network flow problems arise, shedding light on their practical significance.

Keywords: Operational research, Network flow, Linear programming problem, Minimum cost flow.

1 | Introduction

The significance of Operations Research (OR) in current human life in a business context cannot be overstated. OR consultants or analysts undertake detailed assessments of company issues, processes, and vulnerabilities, leveraging advanced pattern and trend analysis to construct models. According to Churchman, C. W [1], "OR has been described as a method, an approach, a set of techniques, a team activity, a combination of many disciplines, an extension of particular disciplines (mathematics, engineering, economics), a new

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discipline, a vocation, even a religion. It is perhaps some of all these things." The field of OR involves using diverse techniques and algorithms to solve particular mathematical models. These models demand finding the values of decision variables that achieve the best possible outcome for the objective function while adhering to a system of inequality and equality constraints. These models yield critical insights and strategic interventions to resolve organizational problems efficiently.

OR plays a pivotal role across various domains. It has various real-life applications, such as the Traveling Salesman problem [2], inventory [3], traffic management [4], network flow optimization [5], Linear Programming (LP) [6], and so on. In the realm of RO, decision-making and problem-solving play crucial roles across diverse industries. Among the topics within RO, Linear Programming Problems (LPPs) emerge as a widely discussed subject, which will be explored in greater detail in the following paragraph.

LP has diverse applications, including different areas but not limited to addressing transportation challenges, optimizing vehicle routes, public transportation schedules, and airline operations to enhance efficiency and reduce costs. Researchers have explored various aspects of LP, such as Tripathi and Kumar [7], which discussed the LPP under uncertainty environments. Later, Tripathi and Kumar [8] and Tripathi et al. [9] conducted a short literature review on the LPP to explore Neutrosophic LP within advanced extended fuzzy contexts.

Additionally, OR contributes to communication network design, improving data transmission efficiency and call center management. Meanwhile, network analysis, which integrates the Critical Path Method (CPM) [10] and Program Evaluation and Review Technique (PERT) [11], offers valuable insights for project management, considering time constraints, resource allocation, and risk analysis. Pratyusha et al. [12] reviewed the extended uncertainty principle for Neutrosophic critical path problems. This first time, Miriyala and Kumar [13] used Python programming in Neutrosophic environments to solve critical path problems in extended uncertain environments. Additionally, Miriyala and Kumar [14] discussed another approach to solving critical path problems in a Neutrosophic environment using Python.

Furthermore, OR models optimize inventory levels, pricing strategies, and the entire supply chain, balancing costs, profits, and service levels. In inventory management, Dubey and Kumar [15] and Dubey et al. [16] reviewed the recent trends and advancements of the inventory model under the extended uncertainty principle. Moreover, with sensitivity analysis, Dubey and Kumar [17] investigated Neutrosophic inventory models in advanced fuzzy contexts.

This initiative centers around achieving specific objectives: enhancing efficiency, reducing costs, and improving overall effectiveness. Noteworthy contributions to the field of Operational Research are summarized in *Table 1* and visualized in *Fig. 1* below.

Table 1. Operational research impact on real-life applications: insights from researchers

Authors	Areas	Applications	Significance
Lapidoth and Moser [18]	Duality theory	Multiple-Antenna Systems	Asymptotic capacity in cost-constrained channels is investigated by the author, with the exploration of capacity bounds through duality and consideration of practical applications for multiple-antenna systems on flat-fading channels.
Fu et al. [19]	Scheduling problem	Grid scheduling problems	The author proposed a novel operator selection framework using Maximum Spread (MS) and Generational Distance (GD) to enhance diversity while maintaining convergence.
Hossein et al. [20]	LPP	Multi-objective optimization problems	The objective of this paper is to expand upon a multi-objective programming methodology by introducing an approach for solving LPP involving grey data and variables.
Zhu et al. [21]	Assignment	Many to Many assignments problem	In this paper, an enhanced approach to address the assignment problem is presented by incorporating backtracking into the Kuhn–Munkres algorithm.

Table 1. Continued.

Authors	Areas	Applications	Significance
Simaiakis and Balakrishnan [22]	Queuing theory	Airport Departure Process	A novel analytical queuing model for airport departure processes is introduced and validated in this paper. The model incorporates stochastic and dynamic features through transient analysis of the D/E/1 queuing system.

The figure labeled as *Fig.1* visually illustrates *Table 1* in a pictorial format.

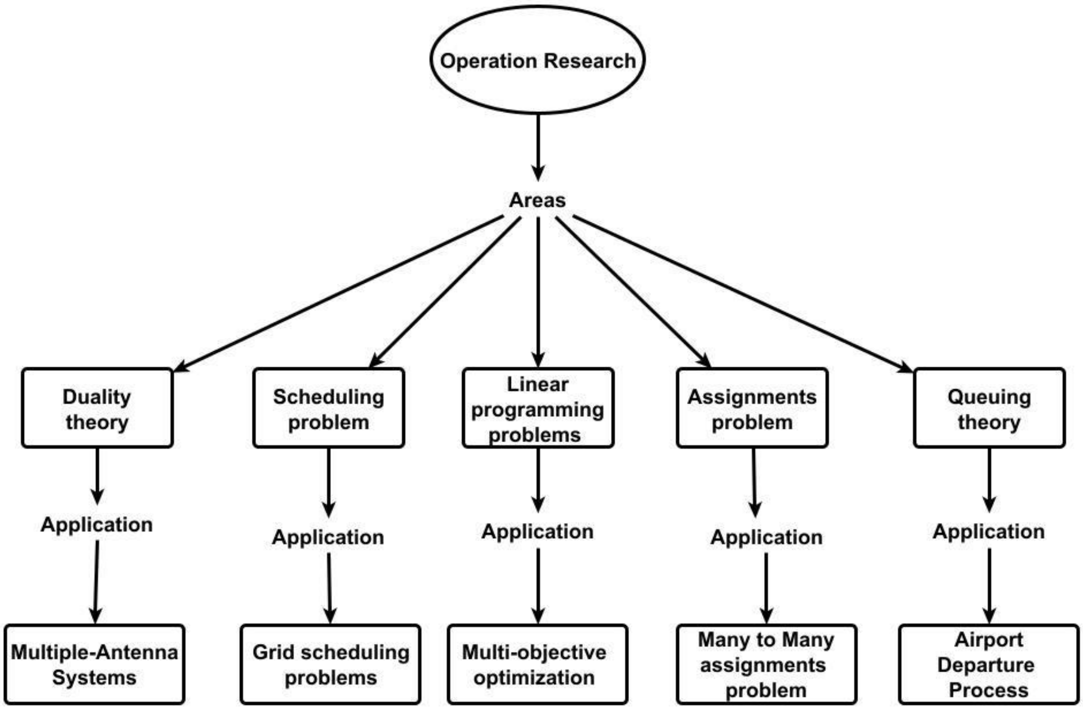

Fig.1. Pictorial representation of Table 1.

LP is an important optimization method. Soviet mathematician Leonid Kantorovich introduced the concept of LP in the late 1930s, followed independently by American mathematician George Dantzig [23]. Its main goal is to find the best possible value for decision variables while meeting all the constraints. These constraints can be things like resource limitations or budget restrictions. Additionally, LP helps us maximize or minimize an objective function representing our ultimate goal.

It has many applications in various fields, including engineering, efficient home energy management [24], finance, transportation, and so on. Furthermore, we have explored significant contributions to the LPP in *Table 2* below.

Table 2. Researchers in various fields have explored the impact of LPPs on real-life applications.

Authors	Year	Methodologies	Applications	Significance
Rusin [25]	1971	Revised simplex method	Quadratic programming	The author presents a computational approach for solving quadratic programming problems by reducing the symmetric basis matrix, akin to the revised simplex method in LP objective functions.
Abaffy and Fodor [26]	2013	ABS Algorithm	Mixed integer LP	The author demonstrates the integration of Gomory's original cutting-plane approach into the ABS class of algorithms.

Table 2. Continued.

Authors	Year	Methodologies	Applications	Significance
Chopra et al. [27]	2017	Hungarian Method	Multirobot assignment problem.	The author suggests a distributed adaptation of the Hungarian method to efficiently tackle the well-known multi-robot assignment.
Darvishi and Nasseri [28]	2020	Dual simplex method	Grey LPPs	To bridge grey system theory with optimization, the author introduces a dual LPP, thereby extending its foundational definitions and concepts.
Singaravelan et al. [24]	2021	Two-Phase Method-based	Efficient Energy Management	The author presents an innovative Two-Phase Simplex Method-based demand response program customized for residential customers, aiming to reduce peak demand, optimize electricity consumption costs, enhance consumer comfort, and improve computational efficiency.

In the survey, we examined the contributions of different authors to LPPs. This underscores the crucial role that LPPs play in the field of OR. Researchers and authors delve into strategies for maximizing profits while minimizing costs and losses within these LPPs. The decision variables in this model represent the quantities to be determined.

However, significant gaps exist in the current understanding of LPP, particularly regarding its application in real-world scenarios. The main objective of this paper is to delve into the contemporary trends and applications of LPPs. Therefore, we aim to bridge these gaps by examining network flow problems. Network problems primarily involve three distinct types of network problems: the shortest path, maximal cost flow, and Minimal Cost Flow (MCF) problems. This review article extensively examines crucial elements concerning network flow problems.

2 | Discuss the Recent Trends and Applications of Linear Programming in Network Flow

Network flow is a valuable application of LPPs that address a wide range of issues. The MCF problem, a subset of network flow problems, represents a significant and practical model. MCF involves demand nodes, supply nodes, and linear flow costs along the edges of the network problems. The primary objective of the MCF problem is to transmit flow from a set of supply nodes to a set of demand nodes through the network arcs, minimizing the total cost. This network consists of supply nodes, demand nodes, and directed arcs, each associated with a cost and capacity constraint.

In this section, we will discuss the MCF problem that stands out as a special case within the broader network flow problem. The MCF problem aims to determine the most cost-effective way to transport a specified flow volume through a network. The task involves finding an optimal solution that meets demand while adhering to the capacity constraints of each arc.

In the discussion, we will explore research conducted by various scholars concerning MCF problems in diverse uncertain environments such as Ghatee and Hashemi [29] explored a new method by using a ranking function, which transforms the fully fuzzified MCF Problem into three crisp problems that can be solved in polynomial time. They provide combinatorial algorithms to find the fuzzy optimal flow. Tripathi and Kumar [30] proposed a new approach to solving MCF problems with Neutrosophic cost problems using a multi-objective LP approach. Ebrahimnejad et al. [31] propose a standardized bounded dual simplex algorithm with penalty costs for solving the MCF problem. Using the proposed method, the minimum cost of carrying a commodity under fuzzy cost through a capacity-constrained network (utilizing available supply at some nodes to meet demand at other nodes) can be determined by using the ranking function. Majumder et al. [32]

developed a new method for a network's ambiguous Maximum Flow (MF), where capacities are modeled as random fuzzy variables. The investigation focuses on developing the Chance-Constrained (CC) model and the Expected Value (EV) model for the MF problem within this fuzzy environment.

Furthermore, we formulate their classical equivalent models. Meanwhile, Tripathi et al. [33] addressed MCF problems with the cost of single-valued Neutrosophic numbers. They introduced a novel approach based on a custom ranking function tailored for the MCF problem-an innovative endeavor within the field of Neutrosophic sets.

Conclusion

In this review, we thoroughly evaluate numerous approaches and methods researchers employ to model and investigate problems. This paper investigates current trends and practical applications of LP in the context of network flow problems. Our study addresses gaps by analyzing recent advancements, focusing on three types of network problems: shortest path, maximal cost flow, and MCF. Furthermore, we explore real-world scenarios where these problems exist. The paper concludes by proposing a tailored research framework suited to the current context and offering recommendations for future research.

Author Contributions

Conceptualization, Tripathi; Methodology, Ranjan Kumar.; Validation, Tripathi and Ranjan kumar; formal analysis bt both of authors.

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Data Availability

Some data were obtained from literature, which has been included in the references. No special permission was required to obtain and use the data.

Conflicts of Interest

The authors declare no conflict of interest. No Funders, other than the authors, played a role in the study's design, in the collection, analysis, or interpretation of the data, in the writing of the manuscript, or in the decision to publish the results.

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