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A New Approach in Fuzzy Optimization of Time-Cost-Quality Trade-Off Problem

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Abstract

Project managers often face different and conflicting objectives when optimizing project resources. In recent years, the demands of project stakeholders regarding reductions in the total cost and time of a project, along with achieving the acceptable quality of the project, have risen significantly. This factor leads researchers to develop models incorporating the quality factor into previously existing time-cost trade-off models. This paper develops a model for the discrete time-cost-quality trade-off problem. For each activity, an execution mode can be selected from a number of possible ones. The time and cost of each mode are assumed to be crisp, but the quality of each mode is a linguistic variable. Therefore, fuzzy logic theory is employed to consider the effects of uncertainty on project quality. Project managers can have different solutions depending on their accepted risk measure by applying α-cut methods in fuzzy logic theory. A new metaheuristic algorithm called NHGA has been developed to solve the model. A case example demonstrates the efficiency of the proposed algorithm for solving the model and its flexibility for project managers' decision-making. The proposed and classic genetic algorithms are prepared using the analysis of variance (ANOVA) method.

Keywords: Time-cost-quality trade-off, Fuzzy theory, Evolutionary algorithms, ANOVA.

1 | Introduction

In construction project scheduling, if it is required to finish the project before a specified due time, the duration of performing some activities must be decreased. To achieve this goal, it is necessary to either increase some resources or change the execution methods of the activities, which will result in increased costs or altered quality. The time-cost-quality trade-off aims to select a set of activities for crashing and an

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appropriate execution method for each activity to minimize the project's cost and time. In contrast, the total project quality is maximized.

In most of the time cost trade-off models, the relationship between the decreased duration and the increased amount of cost is assumed to be linear. Furthermore, the aim is to finish the project before the due time and minimize the total cost. Numerous methods have been proposed [1]-[6] to solve such a linear model. Nonlinear models have also been proposed for the time cost trade-off problem [7]-[10]. The Discrete Time-Cost Trade-Off Problem (DTCTP) applies to real-world applications. Unlike linear models, there are few studies regarding DTCTP. Therefore, a discrete time-cost relationship is more relevant to real-world projects since resources, execution methods, and technology types in real-world projects are represented by discrete values.

The solution methods of DTCTP are classified as exact and heuristic. None of the precise techniques can solve real-world projects with many activities. DTCTP is known as an NP-hard problem [11]. Thus, no exact solution method can be found to have the required efficiency for solving DTCTP. However, many approaches are based on heuristic algorithms for solving DTCTP [12]-[16].

The total project quality is affected by the project crashing. Thus, it is necessary to include the quality factor in the time-cost trade-off problem, leading to the time-cost- quality trade-off problem. This problem is represented by a Discrete-Time Cost Quality Trade-off Problem (DTCQTP) for discrete cases. In this problem, each project activity can be executed in one of many modes. The execution mode of any activity is related to the resources, execution methods, and execution technology used in the activity.

The time-cost-quality trade-off optimization problem aims to complete the project simultaneously, considering the minimal cost, duration, and maximum quality. Not many studies on this optimization problem can be found in the literature. By relaxing some non-realistic assumptions, more realistic methods must be presented. In this paper, we consider the problem in the discrete case (i.e., DTCQTP) and relax the assumption of the linear relationship between time-cost and time-time-quality [17]-[22].

In the operations research literature, many scholars and researchers incorporate fuzzy variables into their models due to the uncertain nature of real-world problems [23]-[26]. In this paper, the quality of each mode is a linguistic variable. Linguistic variables are not numbers but words or sentences in a natural or artificial language. Fuzzy set theory is applied to manage uncertain and linguistic data. Fuzzy numbers are a particular type of fuzzy set, which are normal and convex. Although these numbers can be described using many special methods and shapes, triangular and trapezoidal shapes are widely best used for solving practical applications.

The α-cut is a commonly used method to connect the principles of fuzzy sets with a collection of crisp sets, which can be fed into most of the existing systems. The α–cut level set of A is defined as follows:

$$A_\alpha = \{(x, \mu_A(x)) \geq \alpha : x \epsilon X\}, \qquad \text{for all } \alpha \in [0,1],$$

where,

X: the range of possible values.

$\mu_A(x)$: the membership function takes values within the [0,1] range, which specifies the degree to which x belongs to the fuzzy set A.

A triangular fuzzy number is shown as $\tilde{A} = (a_1, a_2, a_3)$ with the following membership function:

$$\mu_{\tilde{A}}(x)=\begin{cases}0, & x\le a_1,\\ \dfrac{x-a_1}{a_2-a_1}, & a_1<x\le a_2,\\ \dfrac{a_3-x}{a_3-a_2}, & a_2<x\le a_3,\\ 0, & x>a_3.\end{cases}$$

What makes this study different from the previous ones is that it considers uncertainty in the quality of each activity and introduces an innovative approach to solving the problem. The high speed of the algorithm and quick convergence of the solutions make this approach suitable for large projects with large numbers of activities.

The quality of each mode and the acceptable quality of a project cannot be measured precisely by the manager. They are linguistic variables and are presented by fuzzy numbers. Therefore, in this paper, fuzzy set theory is employed to consider the effects of uncertainty on total project quality.

The degree of acceptable risk by the managers is represented as α. The project manager's selection of an appropriate value of α influences the project's outcome significantly. As a result, identifying the relationship between the value of α (0 to 1) and the corresponding risk level (no risk to full risk) by collecting managers' risk attitudes would be indispensable. By setting α equal to zero, no risk is acceptable, and the total quality is considered, while α equal to 1 results in accepting full risk, and project quality is transformed from a fuzzy value into a crisp one.

This paper is organized as follows. In Section 2, we present the problem definition and the problem formulation. In Section 3, a solution procedure is introduced. We developed an algorithm called NHGA. Some examples are presented in Section 4 to illustrate the proposed approach.

2| Problem Modelling

A project is represented by a direct acyclic graph G=(V, E), with m nodes and n arcs where V={ 1,2,3,….,m} is the set of node and E={ (i,j),…,(l,m) } is the set of direct arcs. Arcs and nodes represent activities and events, respectively.

Each project activity says $(i,j)\in E$ can be executed by a set of modes, Mij. Each $k\epsilon M_{ij}$ needs an execution time of tijk, and a cost of cijk, while its quality is represented by $\tilde{q}_{ijk}$. Let k and r be two modes for activity (i,j) and k<r, then, it is assumed tijk>tijr, cijk<cijr, and $\tilde{q}_{ijk}\neq\tilde{q}_{ijr}$. Although in the literature, it is assumed that any decreased activity time leads to decreased activity quality, it is noteworthy that, in real-world projects, this is not always the case. For instance, a new technology can be employed to reduce the required time, while an increase in quality and cost can accompany this reduction.

This research aims to obtain the optimal combination (tijk, cijk, $\tilde{q}_{ijk}$) of each activity for crashing the project network, such that along with reducing the total project duration, the total project cost (direct plus indirect) is minimized. In contrast, the total fuzzy quality of the project does not fall below the desired level. Notations used for the problem formulation are as follows:

C_I: project indirect cost per unit of time.

M_{ij}: set of available execution modes for activity ij; $ij\in E$.

c_{ijk}: direct cost of activity ij if performed by execution mode k.

t_{ijk}: duration of activity ij if performed by execution mode k.

$\tilde{q}_{ijk}$: fuzzy quality of activity ij if performed by execution mode k.

w_{ij}: quality weight of activity ij in the project $\sum_{ij\in E} w_{ij}=1$.

$\widetilde{Q}_L$: lower bound for fuzzy quality of the project.

$y_{ijk} = \begin{cases} 1, & \text{if mode k is assigned to activity ij,} \\ 0, & \text{otherwise.} \end{cases}$

x_i = earliest time of event i, $i = \{1,2, \dots \dots \dots, m\}$.

C: total project cost (direct plus indirect).

T: total project duration.

The problem of DTCQTP is formulated using the mixed integer programming model.

$$\text{Minimize } C = \left(\sum_{ij \in E} \sum_{k \in M_{ij}} c_{ijk} * y_{ijk}\right) + C_I * (x_m - x_1). \quad (1)$$

$$\text{Minimize } T = x_m - x_1. \quad (2)$$

s.t.

$$\sum_{ij \in E} w_{ij} \sum_{k \in M_{ij}} \tilde{q}_{ijk} * y_{ijk} \geq \widetilde{Q}_L. \quad (3)$$

$$x_j - x_i \geq \sum_{k \in M_{ij}} t_{ijk} * y_{ijk}, \quad ij \in E, \quad i, j \in V. \quad (4)$$

$$\sum_{k \in M_{ij}} y_{ijk} = 1, \quad ij \in E. \quad (5)$$

$$x_i \geq 0, \quad i \in V. \quad (6)$$

$$y_{ijk} \in \{0, 1\}, \quad ij \in E, \quad k \in M_{ij}. \quad (7)$$

Eq. (1) and *Eq. (2)* minimize the total project cost and duration. *Eq. (3)* enforces that the fuzzy total project quality does not fall below the acceptable level. *Eq. (4)* preserves the precedence relations between project activities. In *Eq. (5)*, only one execution mode is assigned to each activity.

3 | Solution Procedure

In the DTCQTP, there are a number of execution modes to select for each activity. If the number of project activities is n and there are k execution modes for each activity to choose from, then there are kn solutions, which results in a very large search space. Therefore, it is necessary to develop an efficient evolutionary algorithm.

A New Hybrid Genetic Algorithm (NHGA) is used to obtain the optimal (or near-optimal) solution [21]. The high speed of the algorithm and quick convergence of the solutions make it suitable for solving the above problem. NHGA is developed with some modifications in GA (*Fig. 1*).

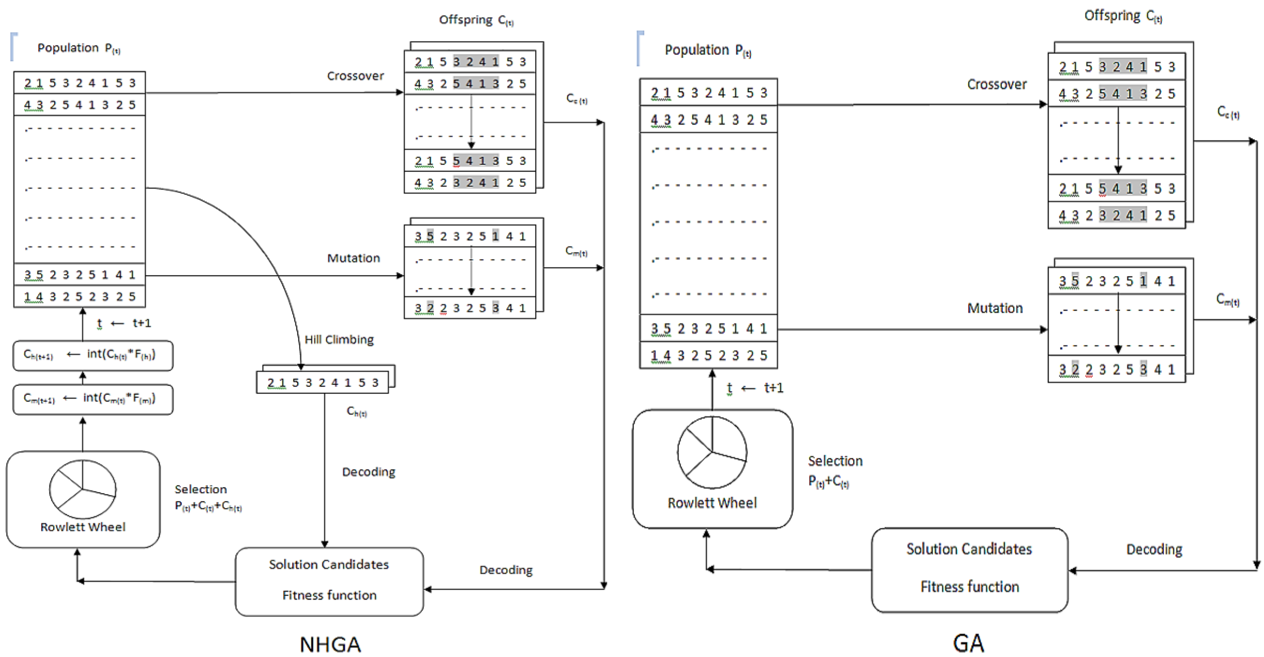

Fig. 1. NHGA and GA flowchart.

3.1| NHGA Implementation

G (V, E) is demonstrated by a node-arc incidence matrix Am×n in which m and n denote the number of nodes and arcs, $A = [a_{ie}]$.

$$a_{ie} = \begin{cases} 1, & \text{if node i starts arc e,} \\ -1, & \text{if node i ends arc e,} \\ 0, & \text{otherwise,} \end{cases}, \qquad i = 1, \dots, m, \qquad e = 1, \dots, n.$$

A chromosome is a set of integer values (genes) representing each activity's modes. In each chromosome, only one mode is selected for each activity, which leads to the combination of $(t,c,\tilde{q})$ for executing the activity. When reading the numbers for all chromosome genes is completed, an execution mode is selected for all project activities, and a chromosome is produced with feasible genes. The main steps of the NHGA are as follows:

Step 1. The data are read, and then N chromosomes with feasible genes are randomly produced as primary solutions. The project data include:

- *Project network matrix ($A_{m\times n}=[a_{ie}]$).*
- *Available execution modes for each activity e and their expected cost, duration, and fuzzy quality (Me and (cek, tek $\tilde{q}_{ek}$)).*
- *The weight of activity e is compared to other project activities (We).*
- *Project indirect cost per unit time (CI).*
- *Lower bound for fuzzy quality of project ($\tilde{Q}L$).*
- *The degree of risk the managers are prepared to take (α).*

The required NHGA parameters include:

- *String size (n).*
- *Number of generation (G).*
- *Population size (N).*
- *Hill climbing rate.*
- *Weight of exponential function (Wexp).*
- *Weight of linear function (Wline).*
- *Two-point crossover rate.*
- *Uniform crossover rate*
- *Mutation rate.*

Step 2. The following notation is used to describe this step:

c_{se}: direct cost of activity e for chromosome S.

t_{se}: duration of activity e for chromosome S.

$\tilde{q}_{se}$: fuzzy quality of activity e for chromosome S.

a_{ie}: the entry of the incidence matrix, as defined before.

b_i: the available supply in node i.

$$l_e = \begin{cases} 1, & \text{if the acivity e is in the path,} \\ 0, & \text{otherwise.} \end{cases}$$

For each chromosome, the total direct cost of project Cd(s), the total project duration T(s), and the fuzzy quality of the project $\tilde{Q}$(s) are calculated as follows:

Total direct cost of project: the sum of direct costs of all project activities.

$$C_{d(s)} = \sum_{e=1}^{n} c_{se}, \qquad s = 1,2, \dots\dots\dots, N. \tag{8}$$

Total project time: the sum of the duration of activities on the critical path.

$$T_{(s)} = \text{Max} \sum_{e=1}^{n} l_e * t_{se}, \qquad s = 1,2, \dots\dots\dots, N. \tag{9}$$

$$\text{s.t.}$$

$$\sum_{e=1}^{n} a_{ie} * l_e = b_i, \qquad i = 1,2, \dots\dots\dots, m, \qquad b_i = \begin{Bmatrix} 1, & \text{if } i = 1, \\ -1, & \text{if } i = m, \\ 0, & \text{otherwise,} \end{Bmatrix}, \tag{10}$$

$$l_e \in \{0, 1\}, \qquad e \in E.$$

Total fuzzy quality of the project: weighted sum of activities quality.

$$\tilde{Q}_{(s)} = \sum_{e=1}^{n} w_e * \tilde{q}_{se}, \qquad s = 1,2, \dots\dots\dots, N. \tag{11}$$

In this paper, the quality of activities ($\tilde{q}_{se}$), is represented as triangular fuzzy numbers, and its α–cut is presented as $(q_{se})_\alpha$. The equations are as follows:

$$(q_{se})_\alpha = \left[q_{se}{}_\alpha^- , \quad q_{se}{}_\alpha^+\right]. \tag{12}$$

$$(w_e * q_{se})_\alpha = \left[w_e * q_{se}{}_\alpha^- , \quad w_e * q_{se}{}_\alpha^+\right]. \tag{13}$$

$$\sum_{e=1}^{n}(w_e * q_{se})_\alpha = \left[\sum_{e=1}^{n} w_e * q_{se}{}_\alpha^- \ , \ \sum_{e=1}^{n} w_e * q_{se}{}_\alpha^+\right], \qquad \text{for all } \alpha \in [0,1]. \tag{14}$$

The calculated fuzzy quality for each chromosome should be checked at this step to ensure it is not lower than the acceptable fuzzy quality ($\widetilde{Q}L$). There are many methods for fuzzy number ranking [27]-[30]. This paper introduces a conservative method for ranking fuzzy numbers by α–cut. This method is as follows:

$$(Q_s)_\alpha = \left[q_{s_\alpha^-}, q_{s_\alpha^+}\right]. \tag{15}$$

$$(Q_L)_\alpha = \left[q_{L_\alpha^-}, q_{L_\alpha^+}\right]. \tag{16}$$

$$\left(q_{s_\alpha^-} \geq q_{L_\alpha^-} \ \wedge \ q_{s_\alpha^+} \geq q_{L_\alpha^+}\right) \Rightarrow (Q_s)_\alpha \geq (Q_L)_\alpha, \qquad \text{for all } \alpha \in [0,1]. \tag{17}$$

Step 3. Determining the fitness function (F(s)) and the probability of selection (P(s)) for each parent chromosome "S" by using the following equations:

$$F_{(s)} = C_{d_{(s)}} + C_I * T_{(s)} - \left(C_{d_{min}} + C_I * T_{min}\right) + \gamma + [\![W_t * C_{d_{min}}/T_{min}]\!] * \left[\left(T_{(s)} - T_{min}\right)/\left(C_{d_{(s)}} - C_{d_{min}} + \gamma\right)\right]. \tag{18}$$

$$P_{(s)} = \frac{\frac{\sum_{s=1}^{N} F_{(s)}}{F_{(s)}}}{\sum_{s=1}^{N}\frac{\sum_{s=1}^{N} F_{(s)}}{F_{(s)}}}, \tag{19}$$

where $C_{d_{min}}$ and T_{min} are minimum direct cost and time of population and W_t is scaling factor. γ is a tiny positive number to prevent dividing by zero in the fitness function, and it also does not permit the fitness function to become zero because the model uses the inverse of the fitness function for the reproduction scale in the proposed NHGA.

For the first objective, *Eq. (1)*, one attempt to obtain the best solution, which is superior to the second objective, *Eq. (2)*. Therefore, *Eq. (18)* is introduced for the fitness function. If $\left(C_{d_{min}} + C_I * T_{min}\right)$ is not present in *Eq. (18)*, then fitness functions will become large numbers, P(s) 's will be so close to each other, and finally, the selection process will lose the necessary efficiency. Because chromosomes with lower F(s) are more desirable, P(s) should be defined so that the lower F(s), the higher the probability of selecting chromosome "S". So, *Eq. (19)* is introduced for P(s).

Step 4. Producing offspring chromosomes from parent chromosomes to enter the next generation is done at this step. Before applying crossover and mutation operators, a small number (c(h)) of the best parents are transferred to the next generation. However, the number of (c(h)) decreases from generation to generation. The decline rate function (F(h)) combines linear and exponential functions with pre-specified weights.

$$F_{(h)} = \frac{w_{exp} * \left(1 - \left(e^{(n_g - G)}\right)\right) + w_{line} * \left(1 - \frac{n_g}{G}\right)}{w_{exp} + w_{line}}, \tag{20}$$

where ng is the generation number index. The direct transfer of the best parents to the next generation with the decline rate function (*Eq. (20)*) is an innovation for improving GA for this problem, which increased the efficiency of the algorithm. After hill climbing, to produce the rest of the offspring (c(t)), crossover (cc(t)) and mutation (cm(t)) operators should be applied. The operators were designed such that, after they are applied, the chromosome genes are still feasible. In this problem, for the number of activities less than 50, a combination of a two-point crossover and a uniform crossover with pre-specified weights was used. Only a uniform crossover was used for the number of activities greater than 50. Uniform crossover has been introduced by [31]. Also, considering the number of activities, one-point to multi-point mutation was use. For instance, in a project with nine activities, two random chromosomes with feasible genes can be as follows:

Parent1= [2,1,5,3,2,4,1,5,3].

Parent2= [4,3,2,5,4,1,3,2,5].

Since the number of activities is small, two-point and uniform crossover and two-point mutation were used in this example. The offspring produced from these parents by applying the operators mentioned above are as follows:

Two-cut-point crossover with random points (e1=4, e2=7),

Offspring1= [2,1,5,5,4,1,3,5,3].

Offspring2= [4,3,2,3,2,4,1,2,5].

Uniform crossover with random mask chromosome [1,1,0,1,0,0,1,0,0],

Offspring1= [4,3,5,5,2,4,3,5,3].

Offspring2= [2,1,2,3,4,1,1,2,5].

Two-point mutation with Random points (e1=6, e2=9),

Offspring1= [2,1,5,3,2,3,1,5,1].

It should be mentioned that the mutation rate (F(m)) is decreasing and uses "*Eq. (21)*" so that in the final generation, the mutation rate will be zero.

$$F_{(m)} = 1 - \frac{n_g}{G}. \tag{21}$$

Step 5. Repeat Steps 2-4 until the chromosomes do not change from one generation to the next.

The steps of NHGA are summarized in *Fig. 1*.

4 | Application Example

As an example, a project that includes nine activities is presented in this section, *Fig. 2*.

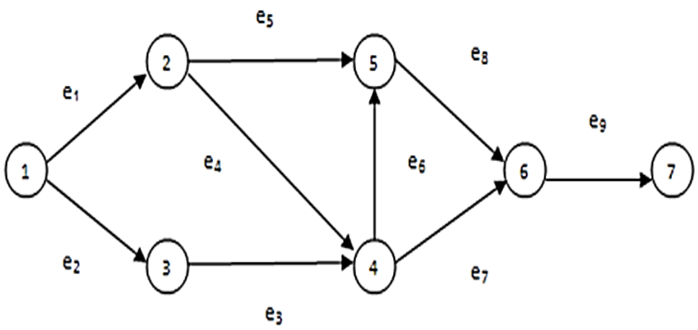

Fig. 2. Project network.

The network matrix is presented in *Fig. 1*. It is an adjacency matrix used to transfer networks to a matrix. Different execution modes of each activity, with associated time, cost, and fuzzy quality, are presented in

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9
1	1	1	0	0	0	0	0	0	0
2	-1	0	0	1	1	0	0	0	0
3	0	-1	1	0	0	0	0	0	0
4	0	0	-1	-1	0	1	1	0	0
5	0	0	0	0	-1	-1	0	1	0
6	0	0	0	0	0	0	-1	-1	1
7	0	0	0	0	0	0	0	0	-1

Table 1. Triangular fuzzy numbers are assumed for any quality of activity. The effect weight of each activity in the total quality (we) is also considered in *Table 1*.

Fig. 3. Network matrix.

The model is programmed in Microsoft Excel using Visual Basic Application (VBA). The project data are entered: *Fig. 1*, *Table 1*, CI= 20, L = [82, 83, 84], and α =1. In this example, there are 1500000 solutions. The proposed model was used to obtain the optimal solution. Since the number of activities is small in the example, a combination of a two-point crossover, a uniform crossover with pre-specified weights, and a two-point mutation were used.

NHGA parameters are set as follows:

G=80, N=50, two-point crossover rate=0.6, uniform crossover rate=0.2, mutation rate=0.2, hill climbing rate =0.15, Wexp=0.4, Wline= 0.6.

The program is running, and the chromosome [4,2,2,1,1,5,1,4,4] is obtained. The output was obtained as its corresponding time, cost, and fuzzy quality (T=34, Cd=1440, = [82.9, 84.5, 86.7]). Also, total project cost was obtained (C=2120). The project manager then obtained other optimum solutions by increasing $\tilde{Q}$L and changing the value for the level of risk (α-cut level), which are presented in the results in *Table 2*.

Table 1. Execution modes of activities.

Mode	Activities	e1	e2	e3	e4	e5	e6	e7	e8	e9
1	T	7	8	8	10	14	8	11	11	11
	C	160	140	110	100	160	130	150	140	150
	Q	90	85	90	88	92	85	87	91	90
2	T	6	7	7	9	13	7	10	10	10
	C	180	150	120	130	170	140	180	150	170
	Q	85	82	85	90	90	82	90	88	88
3	T	5	6	6	8	12	6	9	9	9
	C	190	170	140	140	180	150	190	160	180
	Q	80	80	84	85	86	80	85	85	85
4	T	4	5	5	7	11	5	8	8	8
	C	200	180	150	150	200	170	200	170	200
	Q	70	75	80	75	70	85	90	75	90
5	T	3	4	4	6	10	4		7	
	C	230	200	170	165	220	190		265	
	Q	85	80	90	80	80	90		85	
6	T					9				
	C					240				
	Q					90				
	Wqi	0.1	0.1	0.14	0.11	0.12	0.15	0.08	0.12	0.08

Table 2. The final output of the program.

Q	Ct	Tt	Cd	Solution Chromosome	Qallow
84.48	2120	34	1440	4 2 2 1 1 5 1 4 4	0-84
86.18	2120	35	1420	3 2 1 1 1 5 1 4 4	85
86.18	2120	35	1420	3 2 1 1 1 5 1 4 4	86
87.48	2120	37	1380	1 1 1 1 1 5 1 4 4	87
88.18	2130	37	1390	2 1 1 1 1 5 1 3 4	88
89.04	2140	39	1360	1 1 1 1 1 5 1 2 4	89
89.4	2150	40	1350	1 1 1 1 1 5 1 1 4	89.2
89.62	2180	40	1380	1 1 1 2 1 5 1 1 4	89.6
89.86	2210	40	1410	1 1 1 2 1 5 2 1 4	89.8

The project manager can make decisions by using the information in *Table 2* and analyzing the internal and external conditions of the project.

5 | Experimental Evaluation

This section evaluates the performance of our proposed NHGA and the classical GA. It is used the Relative Deviation Index (RDI) as a common performance measure to compare these algorithms that are computed by

$$\mathrm{RDI} = \frac{C_{alg} - C_{exa}}{C_{exa}} + w_t * \frac{T_{alg} - T_{exa}}{T_{exa}},$$

where Calg and Talg are the total project cost and duration for a given algorithm, respectively, and Cexa and Texa are the solutions obtained using the exact methods for the given small example. The NHGA and GA are implemented with the same parameters twenty times. Their results are analyzed via the analysis of variance (ANOVA) method. The main hypotheses, containing normality, homogeneity of variance, and independence of residuals, are checked, and there is no bias for questioning the experiment's validity. The means plot and Least Significant Difference (LSD) interval for the NHGA and GA are shown in *Fig. 4*. It demonstrates that the NHGA gives better outputs than the GA statistically.

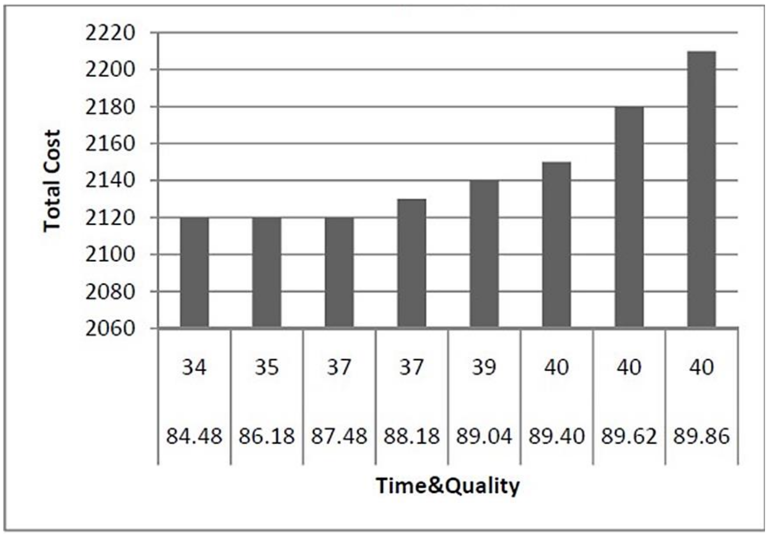

Fig. 4. Time-cost-quality trade-off analysis.

The weight of the exponential function (Wexp) and the weight of the linear function (Wline) is an essential parameter of the decline rate function (F(h)). The considered levels of these parameters (Wexp, Wline) are as (0,1),(0.2,0.8), (0.4,0.6), (0.5,0.5),(0.6,0.4),(0.8,0.2), and (1,0). The NHGA is implemented twenty times for each level of the Wexp. The results are analyzed using the ANOVA method. The means plot and LSD interval for the levels of the Wexp are shown in *Fig. 4*. This figure demonstrates that Wexp = 0.4 or (Wexp, Wline) = (0.4,0.6) results in better outputs than the other (Wexp, Wline) levels statistically.

6| Conclusion

Many project managers perform project crashes intending to reduce total cost and time and achieve a desirable quality. A linguistic variable expresses the quality of each activity. The model proposed in this paper addresses real-world projects. In order to be as realistic as possible, the problem was considered in the discrete case (DTCQTP), and the quality of activities was presented using triangular fuzzy numbers. In this problem, each project activity can be executed in one of several modes. Associated with each execution mode of any activity, there are specific resources, execution methods, and technology. For each mode of an activity, there is a triple combination $(t,c,\tilde{q})$ indicating the time, cost, and fuzzy.

Solving the problem gave an optimal solution, including the time, cost, and project quality for each α-cut level. The α represents the risk the managers are prepared to take. An optimal solution is obtained for each acceptable project quality and α-cut level. The α represents the degree of risk the managers are prepared to accept. Project managers can make effective decisions with these optimal solutions and by analyzing the environmental conditions.

A hybrid algorithm, NHGA, was also developed to solve the problem. The proposed algorithm's high speed and quick convergence make it desirable for large projects with many activities. Furthermore, using the ANOVA method, we used the RDI measure to compare the performance of the NHGA and GA. We have also demonstrated that (Wexp, Wline) = (0.4,0.6) has resulted in statistically better output than the other (Wexp, Wline) levels.

Considering uncertainty in duration time, cost factors, and even simultaneous uncertainty in more than one factor, this model can be extended to such cases, which makes it more realistic.

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