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Solving Vendor Selection Problem by Interval Approximation of Piecewise Quadratic Fuzzy Number

Hamiden Abd El-Wahed Khalifa[1,*], Darko Bozanic[2], Hashem Saberi Najafi[3], Pavan Kumar[4]

[1] Department of Operations Research, Graduate School for Statistical Research, Cairo University, Giza, Egypt; hamiden@cu.edu.eg.

[1] Department of Mathematics, Faculty of Arts and Sciences, Al-Badayeh, Qasim University, Saudi Arabia; Ha.Ahmed@qu.edu.sa

[2] University of Defense in Belgrade, Serbia; dbozanic@yahoo.com

[3] Department of Applied Mathematics, Ayandegan Institute of Higher Education, Tonekabon, Iran; hnajafi@aihe.ac.ir

[4] Department of Mathematics, Faculty of Advanced Languages and Sciences, VIT University Bhopal, Sehore, MP-466114, India; pavankmaths@gmail.com

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Abstract

In this paper, a Vendor Selection Problem (VSP) with fuzzy parameter uncertainty is introduced. The buyer gives a quantity order for a commodity among a set of suppliers. Buyer's objective is to obtain the requirements of lead time, service level and aggregate quality at the minimum cost. Some of the problem parameters are characterized by the piecewise quadratic fuzzy numbers. In addition, an interval approximation for piecewise quadratic fuzzy numbers is proposed for solving the VSP. The VSP with close interval approximation is approached by taking the minimum and maximum values inequalities with the constraints transformed it to two classical Linear Programming Problems (LPPs). The optimal solution for these LPPs is obtained. A numerical example is provided for illustration of the suggested approach.

Keywords: Close interval approximation, Fuzzy optimal solution, Piecewise quadratic fuzzy numbers, Vendor selection.

1 | Introduction

In the literature, Vendor Selection Problem (VSP) was studied by Dickson [1]. First, Zadeh [2] proposed the philosophy of fuzzy sets. Fuzzy decision-making was developed by Zimmermann [3], where they introduced fuzzy programming and LPP with multiple objective functions. Later, several researchers worked on various

Corresponding Author: hamiden@cu.edu.eg

applications based on fuzzy uncertainty. Dubois and Prade [4] investigated the theory and applications of fuzzy uncertainty. Kaufmann and Gupta [5] studied several fuzzy mathematical models with their applications to engineering and management sciences. Maleki et al. [6] proposed a very effective method to solve a LPP including the fuzzy variables and the comparison of fuzzy numbers.

Many authors [7]–[9], studied the problems where all the parameters are uncertain. Lai and Hwang [10] assumed that the parameters might be characterized by triangular possibility distribution; then, they proposed an auxiliary model, which was solved by applying the multi-objective LPP.

Weber and Current [11] studied the multiple objective methods for solving the VSP. Several authors used fuzzy multi-objective approaches to VSP, like [12]–[15], studied the manufacturing delivery performance problem with an application to supply chain management, further investigated by Carvalho and Costa [16].

Several researchers studied the modified S-curve membership function in VSP, for example, Vasant, M. [17], Madroñero [18], Torabi and Hassini [19] presented a novel method based on possibilistic programming to solve a multi-objective supply chain model. Selim and Ozkarahan [20] addressed a supply chain model integrated with network design, which an interactive fuzzy goal programming method solved. Many papers integrated fuzzy and stochastic uncertainty with VSP to determine the performance of suppliers and choose the suitable supplier are those of [21]–[24], and many more. Several papers have been published in green supplier selection in the last few decades. For instance, Jain et al. [25] studied a buyer-initiated decision-making process with the inclusion of the green supplier selection concept.

Luthra et al. [22] investigated a method to solve the sustainable VSP further extended by Chen and Zou [26]. Ehsan et al. [10] studied the green VSP using a MODM algorithm. Ghaniabadi and Mazinani [27] presented a dynamic lot size model for multiple suppliers. Turk et al. [28] studied the multi-objective model in inventory management with supplier selection. Aouadni et al. [29] presented a detailed survey on VSP as well as order allocation problems. This study investigates the approximation of close interval for piecewise quadratic fuzzy to solve the VSP. The remaining portion of the work is arranged as follows. Section 2 recalls some basic concepts needed. Section 3 introduces VSP as an LPP. Section 4 formulates the piecewise quadratic fuzzy VSP. Section 5 provides a numerical example to support and validate the proposed methodology. In the end, a few concluding remarks are reported in Section 6.

2 | Basic Concepts

In this section, some basic terms and results are recalled.

Definition 1 ([30]). A piecewise quadratic fuzzy number (PQFN) is denoted by $\tilde{a}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$, where $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ are reals, is defined by its membership function $\mu_{\tilde{a}_{PQ}}$ as follows (*Fig. 1*).

$$\mu_{\tilde{a}_{PQ}} = \begin{cases} 0, \quad x < a_1, \\ \frac{1}{2}\frac{1}{(a_2-a_1)^2}(x-a_1)^2, \quad a_1 \leq x \leq a_2, \\ \frac{1}{2}\frac{1}{(a_3-a_2)^2}(x-a_3)^2+1, a_2 \leq x \leq a_3, \\ \frac{1}{2}\frac{1}{(a_4-a_3)^2}(x-a_3)^2+1, \quad a_3 \leq x \leq a_4, \\ \frac{1}{2}\frac{1}{(a_5-a_4)^2}(x-a_5)^2, a_4 \leq x \leq a_5, \\ 0, \ x > a_5. \end{cases} \tag{1}$$

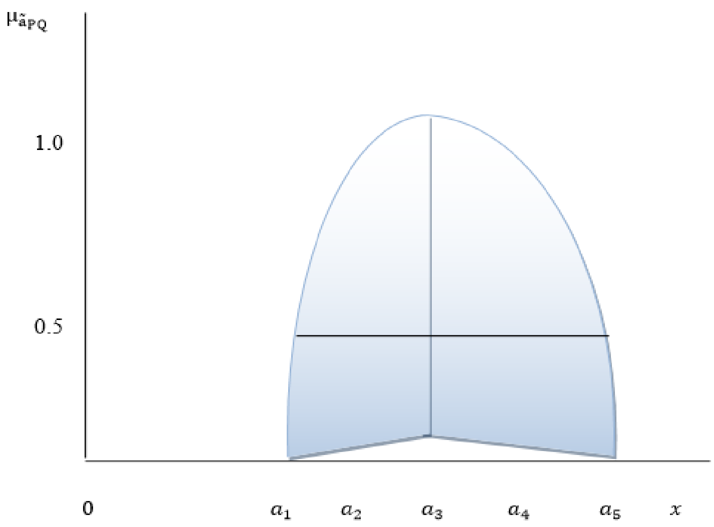

Fig. 1. Graph of a PQFN.

The interval of confidence at level α for the PQFN is defined as

$$(\tilde{a}_{PQ})_\alpha = L_\alpha(\tilde{a}_{PQ}) = [a_1 + 2(a_2 - a_1)\alpha, a_5 - 2(a_5 - a_4)\alpha] \text{ for all } \alpha \in [0,1].$$

Definition 2 ([30]). An interval approximation $[A] = [a_\alpha^L, a_\alpha^U]$ of a PQFN $\tilde{A}$ is referred to as a closed interval approximation if

$$a_\alpha^L = \inf\{x \in \mathfrak{R}: \mu_{\tilde{A}} \geq 0.5\}, \text{ and } a_\alpha^U = \sup\{x \in \mathfrak{R}: \mu_{\tilde{A}} \geq 0.5\}.$$

Definition 3 ([30]). The associated ordinary numbers of PQFN corresponding to $[A] = [a_\alpha^L, a_\alpha^U]$ is $[A] = [a_\alpha^L, a_\alpha^U]$ is equal to $\hat{A} = \frac{a_\alpha^L + a_\alpha^U}{2}$.

Definition 4 ([30]). Suppose that $\tilde{a}_{PQ} = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{b}_{PQ} = (b_1, b_2, b_3, b_4, b_5)$ be two PQFNs. Then

I. Addition: $\tilde{a}_{PQ} \oplus \tilde{b}_{PQ} = (a_1 + b_1, a_2 + a_2, a_3 + b_3, a_4 + b_4, a_5 + b_5)$.

II. Scalar multiplication: $\alpha.\tilde{a}_{PQ} = (\alpha\, a_1, \alpha\;\; a_2, \alpha\, a_3, \alpha\, a_4, \alpha\, \alpha a_5), \alpha > 0$.

Definition 5 ([30]). Let $[A] = [(a_\alpha)^L, (a_\alpha)^U]$, and $[B] = [(b_\alpha)^L, (b_\alpha)^U]$ be two interval approximations of PQFN. Then, two define the arithmetic operations as follows:

I. Addition: $[A] \oplus [B] = [A] \oplus [B] = [a_\alpha^L + b_\alpha^L, a_\alpha^U + b_\alpha^U]$.

II. Subtraction: $[A] \ominus [B] = [a_\alpha^L - b_\alpha^U, a_\alpha^U - b_\alpha^L]$.

III. Scalar multiplication: $\alpha\,[A] = \begin{cases} [\alpha\, a_\alpha^L, \alpha\, a_\alpha^U], \alpha > 0, \\ [\alpha\, a_\alpha^U, \alpha\, a_\alpha^L], \alpha < 0. \end{cases}$

IV. Multiplication: $[A] \odot [B] = \left[\frac{a_\alpha^U b_\alpha^L + a_\alpha^L b_\alpha^U}{2}, \frac{a_\alpha^L b_\alpha^L + a_\alpha^U b_\alpha^U}{2}\right]$.

V. Division: $\frac{[A]}{[B]} = \begin{cases} \left[\frac{2a_\alpha^L}{b_\alpha^L + b_\alpha^U}, \frac{2a_\alpha^U}{b_\alpha^L + b_\alpha^U}\right], [B] > 0 \text{ and } b_\alpha^L + b_\alpha^U \neq 0, \\ \left[\frac{2a_\alpha^U}{b_\alpha^L + b_\alpha^U}, \frac{2a_\alpha^L}{b_\alpha^L + b_\alpha^U}\right], [B] < 0 \text{ and } b_\alpha^L + b_\alpha^U \neq 0. \end{cases}$

VI. The order relations $(\approx, \cong, \lesssim, \gtrsim)$ are defined as

I. Fuzzy equal: $[A] \approx [B]$ if $a_\alpha^L = b_\alpha^L$ and $a_\alpha^U = b_\alpha^U$.

II. Fuzzy equivalent: $[A] \cong [B]$ if $a_\alpha^L + a_\alpha^U = b_\alpha^L + b_\alpha^U$.

III. $[A] \lesssim [B]$ if $a_\alpha^L \leq b_\alpha^L$ and $a_\alpha^U \leq b_\alpha^U$, or $a_\alpha^L + a_\alpha^U \leq b_\alpha^L + b_\alpha^U$.

IV. $[A] \gtrsim [B]$ if $a_\alpha^L \geq b_\alpha^U$ and $a_\alpha^U \geq b_\alpha^L$.

3 | Vendor Selection Problem

A VSP is defined as a LPP for minimizing price under the restriction of quality performance measures, services, and lead time is provided as [21]

$$\min Z(x) = \sum_{i=1}^{n} p_i x_i.$$

Subject to **(1)**

$$\sum_{i=1}^{n} q_i x_i \geq Q,$$
$$\sum_{i=1}^{n} s_i x_i \geq S,$$
$$\sum_{i=1}^{n} l_i x_i \leq L,$$
$$\sum_{i=1}^{n} x_i = 1,$$
$$x_i \geq 0 \text{ for all } i.$$

Where

x_i: fraction of demand allocated to vendor i.

p_i: price of item i.

q_i: quality level of item i.

l_i: lead time of item i.

s_i: level of service of item i.

Q: required overall quality level.

L: lead time level.

S: service level.

4 | Piecewise Quadratic Fuzzy VSP

Consider the following PQFVS using the close interval approximation as

$$\min Z(x) = \sum_{i=1}^{m} [p_i] x_i.$$

Subject to **(2)**

$$\sum_{i=1}^{m} [q_i] x_i \geq [Q],$$
$$\sum_{i=1}^{m} [s_i] x_i \geq [S],$$
$$\sum_{i=1}^{m} [l_i] x_i \leq [L],$$
$$\sum_{i=1}^{m} x_i = 1,$$
$$x_i \geq 0 \text{ for all } i.$$

Where

$[l_i] = [(l_i)_\alpha^L, (l_i)_\alpha^U],$
$[p_i] = [(p_i)_\alpha^L, (p_i)_\alpha^U],$
$[q_i] = [(q_i)_\alpha^L, (q_i)_\alpha^U],$
$[s_i] = [(s_i)_\alpha^L, (s_i)_\alpha^U],$
$[L] = [L_\alpha^L, L_\alpha^U],$
$[Q] = [Q_\alpha^L, Q_\alpha^U],$
$[S] = [S_\alpha^L, S_\alpha^U] \in P(\mathfrak{R}).$

$P(\mathfrak{R})$ = set of all close interval approximations on $\mathfrak{R}$.

Definition 6. Any x_j which satisfies the constraints in *Problem (2)* is referred to as a feasible solution. Let G be the set of all feasible solutions of *Problem (1)*. We claim that $x^* \in G$ is said to be an optimal solution only when $[p]x^* \leq [p]x$ for all $x \in G$.

Lemma 1. *Problems (1)* and *(2)* are equivalent.

Proof: let G_1 and G_2 denote the respective two feasible solution sets of *Problems (1)* and *(2)*. Then, an element $x \in G_1$ if and only if

$$\sum_{I=1}^{n} [q_i]x_i \geq [Q],$$
$$\sum_{i=1}^{n} [s_i]x_i \geq [S],$$
$$\sum_{i=1}^{n} [l_i]x_i \leq [L],$$
$$\sum_{i=1}^{m} x_i = 1.$$

If and only if

$$\sum_{I=1}^{m} [(q_i)_\alpha^L, (q_i)_\alpha^U]x_i \geq [Q_\alpha^L, Q_\alpha^U],$$
$$\sum_{i=1}^{m} [(s_i)_\alpha^L, (s_i)_\alpha^U]x_i \geq [S_\alpha^L, S_\alpha^U],$$
$$\sum_{i=1}^{m} [(l_i)_\alpha^L, (l_i)_\alpha^U]x_i \leq [L_\alpha^L, L_\alpha^U],$$
$$\sum_{i=1}^{m} x_i = 1.$$

If and only if

$$\sum_{I=1}^{m} [(q_i)_\alpha^L x_I, (q_i)_\alpha^U x_i] \geq [Q_\alpha^L, Q_\alpha^U],$$
$$\sum_{i=1}^{m} [(s_i)_\alpha^L x_i, (s_i)_\alpha^U x_i] \geq [S_\alpha^L, S_\alpha^U],$$
$$\sum_{I=1}^{m} [(l_i)_\alpha^L x_I, (l_i)_\alpha^U x_i] \leq [L_\alpha^L, L_\alpha^U],$$
$$\sum_{i=1}^{m} x_i = 1.$$

If and only if

$$\sum_{I=1}^{m}\left(\frac{(q_i)_\alpha^L + (q_i)_\alpha^U}{2}\right)x_i \geq \left(\frac{Q_\alpha^L + Q_\alpha^U}{2}\right),$$
$$\sum_{i=1}^{m}\left(\frac{(s_i)_\alpha^L + (s_i)_\alpha^U}{2}\right)x_i \geq \left(\frac{S_\alpha^L + S_\alpha^U}{2}\right),$$
$$\sum_{I=1}^{m}\left(\frac{(l_i)_\alpha^L + (l_i)_\alpha^U}{2}\right)x_i \leq \left(\frac{L_\alpha^L + L_\alpha^U}{2}\right),$$
$$\sum_{i=1}^{m} x_i = 1.$$

If and only if

$$\sum_{i=1}^{m} q_i x_i \geq Q\,,$$
$$\sum_{i=1}^{m} s_i x_i \geq S,$$
$$\sum_{i=1}^{m} l_i x_i \leq L,$$
$$\sum_{i=1}^{m} x_i = 1.$$

If and only if

$x \in G_2$.

Hence

$G_1 \cong G_2$.

Let X^* be the optimal feasible solution of *Problem (2)*, then we obtain.

$[P]X^* \leq [P]X$ for all $X \in G_1$.

If and only if

$$\sum_{i=1}^{m}[p_i]x_i^* \leq \sum_{i=1}^{m}[p_i]x_i \text{ for all } x_j \in G_1,$$

If and only if

$$\sum_{i=1}^{m}\left(\frac{(p_i)_\alpha^L + (p_i)_\alpha^U}{2}\right)x_i^* \leq \sum_{i=1}^{m}\left(\frac{(p_i)_\alpha^L + (p_i)_\alpha^U}{2}\right)x_i,$$

If and only if

$$\sum_{i=1}^{m} p_i x_i^* \leq \sum_{i=1}^{n} p_i x_i\,.$$

Thus, we have X^* is an optimal feasible solution of the crisp VSP as given by (1).

Referring to the interval arithmetic operations [31] *Problem (2)* is converted into 2^{m+1} inequalities as follows:

$$\sum_{i=1}^{m}(q_i)_\alpha^L x_i \geq Q_\alpha^U, \quad \sum_{i=1}^{m}(q_i)_\alpha^U x_i \geq Q_\alpha^L\,,$$
$$\sum_{i=1}^{m}(s_i)_\alpha^L x_i \geq S_\alpha^U, \quad \sum_{i=1}^{m}(s_i)_\alpha^U x_i \geq S_\alpha^L,$$
$$\sum_{i=1}^{m}(l_i)_\alpha^L x_i \leq L_\alpha^L, \quad \sum_{i=1}^{m}(l_i)_\alpha^U x_i \leq L_\alpha^U,$$
$$\sum_{i=1}^{m} x_i = 1,$$

$x_i \geq 0$ for all i.

Let G_i be a set of solutions to the i inequality and define.

$\overline{G} = \bigcup_{i=1}^{2m+1} G_i$ and $\underline{G} = \bigcap_{i=1}^{2m+1} G_i$.

Definition 7. Suppose that: $\sum_{i=1}^{m}\left[q_1^{(i)}, q_2^{(i)}\right]x_i \geq [Q_1, Q_2]$, $\sum_{i=1}^{m}\left[s_1^{(i)}, s_2^{(i)}\right]x_i \geq [S_1, S_2]$ and $\sum_{i=1}^{m}\left[l_1^{(i)}, l_2^{(i)}\right]x_i \geq [L_1, L_2]$. Then inequalities $\sum_{i=1}^{m} q^{(i)}x_i \geq Q$, $\sum_{i=1}^{m} s^{(i)}x_i \geq S$ and $\sum_{i=1}^{m} l^{(i)}x_i \geq L$ are called the characteristic formulas of $\sum_{i=1}^{m}\left[q_1^{(i)}, q_2^{(i)}\right]x_i \geq [Q_1, Q_2]$, $\sum_{i=1}^{m}\left[s_1^{(i)}, s_2^{(i)}\right]x_i \geq [S_1, S_2]$ and $\sum_{i=1}^{m}\left[l_1^{(i)}, l_2^{(i)}\right] \geq [L_1, L_2]$; respectively. Here, $q^{(i)} \in \left[q_1^{(i)}, q_2^{(i)}\right]$, $s^{(i)} \in \left[s_1^{(i)}, s_2^{(i)}\right]$, and $l^{(i)} \in \left[l_1^{(i)}, l_2^{(i)}\right]$.

Definition 8. For each inequality $\sum_{i=1}^{m}\left[q_1^{(i)}, q_2^{(i)}\right]x_i \geq [Q_1, Q_2]$, $\sum_{i=1}^{m}\left[s_1^{(i)}, s_2^{(i)}\right]x_i \geq [S_1, S_2]$ and $\sum_{i=1}^{m}\left[l_1^{(i)}, l_2^{(i)}\right]x_i \geq [L_1, L_2]$ if there is one characteristic formula such that its solution set is similar to $\overline{G}$ or $\underline{G}$, in this case, we say that the characteristic formulas as maximum and minimum values range inequalities.

Proposition 1. Let the following inequalities $\sum_{i=1}^{m}\left[q_1^{(i)}, q_2^{(i)}\right]x_i \geq [Q_1, Q_2]$, $\sum_{i=1}^{m}\left[s_1^{(i)}, s_2^{(i)}\right]x_i \geq [S_1, S_2]$ and $\sum_{i=1}^{m}\left[l_1^{(i)}, l_2^{(i)}\right]x_i \geq [L_1, L_2]$. Then, we have $\sum_{i=1}^{m}(q_i)_\alpha^L x_i \geq Q_\alpha^U$, $\sum_{i=1}^{m}(q_i)_\alpha^U x_i \geq Q_\alpha^L$, $\sum_{i=1}^{m}(s_i)_\alpha^L x_i \geq S_\alpha^U$, $\sum_{i=1}^{m}(s_i)_\alpha^U x_i \geq S_\alpha^L$, $\sum_{i=1}^{m}(l_i)_\alpha^L x_i \leq L_\alpha^L$, $\sum_{i=1}^{m}(l_i)_\alpha^U x_i \leq L_\alpha^U$ are maximum and minimum values range inequalities for these constraint conditions, respectively.

By the arithmetic operations of intervals, *Problem (2)* is reduced into the following problems as

$$\min Z(x) = \sum_{i=1}^{m} (p_i)_\alpha^L x_i.$$

Subject to **(3)**

$$\sum_{i=1}^{m} (q_i)_\alpha^U x_i \geq Q_\alpha^L,$$

$$\sum_{i=1}^{m} (q_i)_\alpha^U x_i \geq S_\alpha^L,$$

$$\sum_{i=1}^{m} (l_i)_\alpha^U x_i \leq L_\alpha^U,$$

$$\sum_{i=1}^{m} x_i = 1,$$

$$x_i \geq 0 \text{ for all i.}$$

Now, we obtain the following optimization problem:

$$\min Z(x) = \sum_{i=1}^{m} (p_i)_\alpha^U x_i.$$

Subject to **(4)**

$$\sum_{i=1}^{m} (q_i)_\alpha^L x_i \geq Q_\alpha^U,$$

$$\sum_{i=1}^{m} (q_i)_\alpha^U x_i \geq S_\alpha^L,$$

$$\sum_{i=1}^{m} (q_i)_\alpha^L x_i \geq S_\alpha^U,$$

$$\sum_{i=1}^{m} (l_i)_\alpha^L x_i \leq L_\alpha^L,$$

$$\sum_{i=1}^{m} x_i = 1,$$

$$x_i \geq 0 \text{ for all i.}$$

Suppose that the optimal solutions to *Problems (3)* and *(4)* are

$x_1^*, x_2^*, \ldots, x_m^*; Z_1^*,\qquad x_1^{**}, x_2^{**}, \ldots, x_m^{**}; Z_2^{**}.$

Thus, the optimal solution to the close interval approximation *Problem (2)* is

$$\min Z(x) = [Z_1^*, Z_2^{**}].$$
$$\begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ . \\ x_m \end{bmatrix} = \begin{bmatrix} [x_1^*, x_1^{**}] \\ [x_2^*, x_1^{**}] \\ . \\ . \\ . \\ [x_m^*, x_m^{**}] \end{bmatrix}. \qquad (5)$$

5 | Numerical Example

Consider the following piecewise quadratic fuzzy VSP as in *Table 1*.

Table 1. Vendor's source piecewise quadratic fuzzy data.

Factor	Supplier			Required Level
	A	B	C	
Quantity	0 to 1	0 to 1	0 to 1	1
Price ($/unit)	(8, 9, 10.5, 11, 12)	(8.5, 10, 10.5, 11.5, 12)	(9, 10.5, 11, 12, 13)	-----
Quality (%)	(75, 80, 90, 95, 105)	(83, 87,94, 97, 100)	(84, 92, 98, 102, 105)	(75, 80, 83, 85, 90)
Lead time (days)	(18, 20, 30, 35, 40)	(15, 20, 25, 30, 35)	(24, 25, 26, 27, 28)	(15, 20, 25, 30, 35)
Service (%)	(75, 80, 90, 95, 100)	(75, 92, 95, 100, 110)	(85, 90, 95, 100, 105)	(70, 75, 78, 80, 100)

The close interval approximations for the problem are presented in *Table 2*.

Table 2. Vendor's source close interval approximation data.

Factor	Supplier			Required Level
	A	B	C	
Quantity	0 to 1	0 to 1	0 to 1	1
Price ($/unit)	[9, 11]	[10, 11.5]	[10.5, 12]	-----
Quality (%)	[80, 95]	[87, 97]	[92, 102]	[80, 85]
Lead time (days)	[20, 35]	[20, 30]	[25, 27]	[20, 30]
Service (%)	[80, 95]	[92, 100]	[90, 100]	[75, 80]

Applying *Problem (2)* to these data

$$\min Z(x) = [9, 11]x_1 \oplus [10, 11.5]x_2 \oplus [10.5, 12]x_3.$$

Subject to (6)

$$[0.80, 0.95]x_1 \oplus [0.87, 0.97]x_2 \oplus [0.92, 1.02]x_3 \geq [0.80, 0.85],$$
$$[0.80, 0.95]x_1 \oplus [0.92, 1.00]x_2 \oplus [0.90, 1.00]x_3 \geq [0.75, 0.80],$$
$$[20, 35]x_1 \oplus [20, 30]x_2 \oplus [25, 27]x_3 \leq [20, 30],$$
$$x_1 + x_2 + x_3 = 1.0,$$
$$x_1, x_2, x_3 \geq 0.$$

The interval *Problem (6)* is reduced to the following two LPPs as follows:

$$\min Z_1(x) = 9x_1 + 10x_2 + 10.5x_3.$$

Subject to (7)

$$0.95x_1 + 0.97x_2 + 1.02x_3 \geq 0.80,$$
$$0.95x_1 + 1.00x_2 + 1.00x_3 \geq 0.75,$$
$$35x_1 + 30x_2 + 27x_3 \leq 30,$$
$$x_1 + x_2 + x_3 = 1.0,$$
$$x_1, x_2, x_3 \geq 0.$$

Equivalently, we have the following problem.

$\min Z_2(x) = 11x_1 + 11.5x_2 + 12x_3.$
Subject to **(8)**
$0.80x_1 + 0.87x_2 + 0.92x_3 \geq 0.85,$
$0.80x_1 + 0.92x_2 + 0.90x_3 \geq 0.80,$
$20x_1 + 20x_2 + 25x_3 \leq 20,$
$x_1 + x_2 + x_3 = 1.0,$
$x_1, x_2, x_3 \geq 0.$

The solutions to the two *Problems (7)* and *(8)* are given below:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} [x_1^*, x_1^{**}] \\ [x_2^*, x_2^{**}] \\ [x_3^*, x_3^{**}] \end{bmatrix} = \begin{bmatrix} [0.2857, 0.375] \\ [0, 0.7143] \\ [0, 0.625] \end{bmatrix} \text{ and } \min Z(x) = [Z_1^*, Z_2^{**}] = [9.9375, 11.3517].$$

6 | Conclusions

The VSP as an application of fully fuzzy LPP has been studied. The VSP with close interval approximation has been approached by taking the minimum and maximum value inequalities with the constraints reduced into two classical LPPs. The presented model possesses great future scope for development. One may consider the time value coefficients for more realistic objective function calculations. In addition, the model can be extended in the form of a multi-stage stochastic programming approach. As another future research direction, one may consider some more efficient algorithms to demonstrate the suggested model.

Conflicts of Interest

The authors declare no conflicts of interest to report regarding the present study.

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